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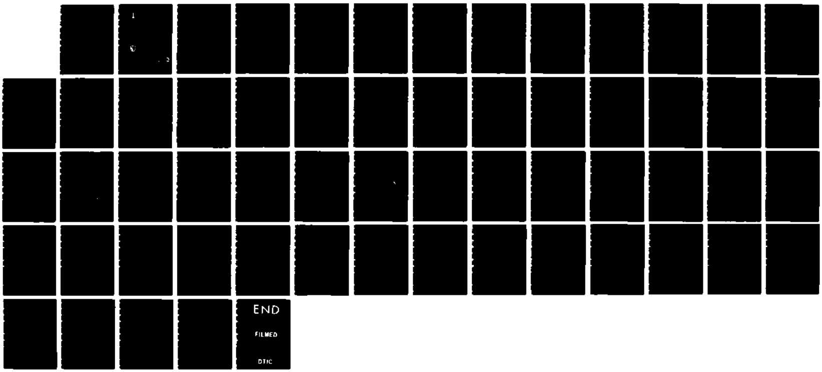

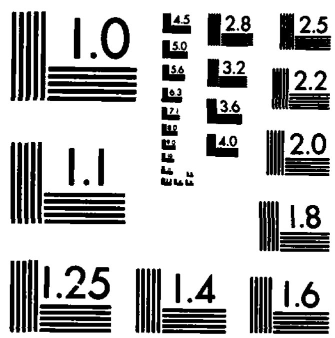

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TECHNICAL REPORT RG-85-5

FURTHER RESULTS FOR APPLICATION OF DISTURBANCE MINIMIZATION CONTROL TECHNIQUES TO A LINEAR, TIME-INVARIANT, SECOND-ORDER STATE SET-POINT REGULATOR PROBLEM

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OCTOBER 1984

U.S. ARMY MISSILE COMMAND
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Further results on The Application of Disturbance Minimizing Control Design Techniques to a linear, time-invariant state set-point regulator are presented using a second-order system as an example. A comparison is presented of performance obtained from a controller designed to minimize steady-state error versus one designed to minimize disturbance effects on error dynamics. Two examples are presented to illustrate regions of positive and negative disturbance utility. It is also shown that control energy expenditure can be

20. Abstract (Continued)

reduced if use is made of external disturbances.

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I. INTRODUCTION

Disturbances are defined as the uncontrollable inputs which act on a dynamical system. There are many varieties of disturbance inputs which can be associated with a controlled system and they are, for the most part, completely unpredictable in magnitude and in their arrival times.

In practice, additive disturbances, i.e., disturbances which are represented by terms added to the plant state equation, can arise from motivating effects external to the plant (external disturbances) or from motivating effects arising from the physical characteristics of plant subsystems or internal plant dynamics (internal disturbances). Further, these disturbances can be divided into two categories: (a) noise disturbances - characterized by random and erratic behavior with relatively high-frequency content and (b) waveform structured disturbances - characterized by a degree of waveform regularity which can be described, piecewise in time, by differential equations forced by sparse sequences of impulses. The nature of these disturbances may be either completedy known (through direct prior or realtime observation or test), completely unknown (random-like), or partially known.

Johnson [1-6, 10] introduced the idea of mathematically describing uncertain waveform-structured disturbances by representing them as a weighted linear combination of known basis functions of the form

$$w(t) = \sum_{i=1}^{n} c_i f_i(t), \tag{1}$$

where w(t) is the plant disturbance vector and is a p-vector and the weighting coefficients c_i are completely unknown constants which can change in magnitude in a random, once-in-a-while, fashion. The basis functions $f_i(t)$ are completely known because they are chosen by the designer based on the waveform patterns exhibited (or thought to be exhibited) by the disturbance.

Johnson [1-11] developed a control engineering design technique, referred to as Disturbance Accommodation, wherein a combination of waveform-mode disturbance modeling and state-variable control methods are utilized to design controllers which will: (1) absorb (counteract), (2) minimize or (3) constructively utilize the effects of uncertain disturbances on the plant. Three main classes of controllers are considered within the overall cognomen of Disturbance Accommodating Control Theory. These are, (1) Disturbance Absorption Controllers (DAC), (2) Disturbance Minimization Controllers (DMC), and (3) Disturbance Utilizing Controllers (DUC). Each class of controller has its own associated design goals and design methodology. The mathematical theories of DAC and DUC were thoroughly developed in [1-12]. The theory and techniques associated with DMC were compiled and extended in [13].

A number of examples were presented in [13] to illustrate the application of various disturbance minimization techniques. This report will present further results on the application of disturbance minimizing control design techniques to linear, time-invariant state set-point regulators.

II. LINEAR DYNAMICAL SYSTEMS

The class of systems to be considered in this report are "linear, time-invariant, dynamical systems", so-called because the vector differential equation for the state x(t) is a linear differential equation, the transformation between the state space and output space is linear, and the elements of the matrices in the plant model are constant with respect to time.

These systems will be represented by equations of the general form

$$\dot{x}(t) = Ax(t) + Bu(t) + Fw(t) \tag{2}$$

$$y(t) = Cx(t) + Eu(t) + Gw(t), \tag{3}$$

where x(t) is the plant state vector and is an n-vector, u(t) is the plant control input vector and is an r-vector, w(t) is the plant disturbance vector and is a p-vector, y(t) is the plant output vector and is an m-vector and A, B, F, C, E, G are appropriate size, known matrices with time-invariant elements. In addition, the general form of the disturbance state model is [10]

$$w(t) = Hz(t) + Lx(t) \tag{4}$$

$$\dot{z}(t) = Dz(t) + Mx(t) + \sigma(t), \tag{5}$$

where z(t) is the ρ- dimensional disturbance state vector, $\sigma(t)$ is a sparsely populated vector impulse sequence and H, L, D, M are appropriate size, known matrices.

III. BACKGROUND

In [13] several methods were presented for minimizing, via direct control action, the effects of constant disturbance components, which are not completely absorbable, on linear, time-invariant state set-point regulators. The metric used for the minimization process is the norm defined by

$$\left\| Ax - b \right\|_Q^2 = (Ax-b)^T Q(Ax-b), \; Q > 0. \tag{6}$$

The design objective in each case is the minimization of the distance between the attainable and desired set-point, where this distance is defined by the Euclidean norm,

$$d^2 = \left\| \varepsilon \right\|_I^2 = \varepsilon^T \varepsilon, \tag{7}$$

of the error vector between x_{sp} and the plant state $x(t)$, i.e.,

$$\varepsilon(t) = x_{sp} - x(t). \tag{8}$$

An expression for the dynamics associated with this error can be derived by differentiating (8) and substituting in the appropriate terms from (2). The result can be expressed as

$$\dot{\varepsilon}(t) = \dot{x}_{sp} - \dot{x}(t) = A\varepsilon(t) - Bu(t) - Ax_{sp} - Fw(t), \tag{9}$$

where Ax_{sp} represents the "set-point disturbance" term.

In disturbance accommodating control design, the control vector $u(t)$ is considered to be an ordered collection of the various independent control inputs which are available to accomplish the primary control objective and to "accommodate" the disturbances which are acting on the system. In the design of disturbance minimization controllers, it is common practice to split (allocate) the total control $u(t)$ into two parts as follows:

$$u(t) = u_p(t) + u_d(t), \tag{10}$$

where $u_p(t)$ is given the task of accomplishing the primary control objective and $u_d(t)$ is given the task of disturbance accommodation. The part $u_d(t)$ can be further subdivided into component vectors, as required. For the methods considered in this report $u_d(t)$ will be allocated as

$$u_d(t) = u_{ds}(t) + u_{dw}(t). \tag{11}$$

The component $u_{ds}(t)$ will be designed to accommodate the effects of the set-point disturbance term while $u_{dw}(t)$ will be designed to accommodate the effects of the external disturbance term. If the plant is completely controllable and is also completely observable, the control $u_p(t)$ can be designed in the form

$$u_p(t) = Kx(t). \tag{12}$$

Given the allocation of the control vector u(t), (9) can be re-written as

$$\dot{\varepsilon}(t) = A\varepsilon(t) - Bu_p(t) - Bu_{ds}(t) - Bu_{dw}(t) - Ax_{sp} - Fw(t). \tag{13}$$

Upon substitution from (4), (13) becomes

$$\dot{\varepsilon}(t) = A\varepsilon(t)-Bu_p(t)-Bu_{ds}(t)-Bu_{dw}(t)-Ax_{sp}-FHz(t)-FLx_{sp}-FL\varepsilon(t). \tag{14}$$

In the case of (14), one would design $u_p(t)$ in the form

$$u_p(t) = -K\varepsilon(t) \tag{15}$$

with K chosen such that the homogeneous system

$$\dot{\varepsilon}(t) = (A+FL+BK)\varepsilon(t) \tag{16}$$

will yield $\varepsilon(t)\to 0$ "rapidly". If one lets $\tilde{A} = A+FL+BK$, then (15) can be expressed as

$$\dot{\varepsilon}(t) = \tilde{A}\varepsilon(t) - ((A + FL)\, x_{sp} + Bu_{ds}(t)) - (FHz(t) + Bu_{dw}(t)). \tag{17}$$

One of the approaches developed in [13] was to provide minimization of ε_{ss} by use of an allocated disturbance control component. If one assumes that a unique steady-state solution exists for $\varepsilon(t)$, i.e., all λ_i of A have negative real parts and all disturbance terms have a limit as time approaches infinity, then ε_{ss} can be found by setting $\dot{\varepsilon}(t)$ in (17) to zero and solving for the ε which satisfies the resulting equation. If this is done, the expression for the steady-state error is found to be

$$\varepsilon_{ss} = \tilde{A}^{-1}\, [(A + FL)\, x_{sp} + Bu_{ds} + FHz_\infty + Bu_{dw}]. \tag{18}$$

One now wishes to design u_{ds} and u_{dw} such that

$$\left\| \varepsilon_{ss1} \right\| = \left\| \tilde{A}^{-1}\, [(A + FL)x_{sp} + Bu_{ds}] \right\| \tag{19}$$

and

$$\left\| \varepsilon_{ss2} \right\| = \left\| \tilde{A}^{-1}\, (FHz_\infty + Bu_{dw}) \right\| \tag{20}$$

are minimized. If Q in (6) is chosen to be the identity matrix I, the resulting minimum norm control components which minimize the Euclidean norms of (19) and (20) were shown ([13]) to be

$$u^*_{ds} = -(\tilde{A}^{-1}\, B)^{\dagger}\, \tilde{A}^{-1}\, (A + FL)x_{sp} \tag{21}$$

and

$$u^*_{dw} = -(\tilde{A}^{-1}\ B\ )^{\dagger}\ \tilde{A}^{-1}\ FHz_{\infty}\ , \tag{22}$$

where $(.)^{\dagger}$ denotes the Moore-Penrose generalized inverse of (.). The minimal norm set-point error ε_{ss} was also shown to be given by

$$\varepsilon^*_{ss} = [I - (\tilde{A}^{-1}\ B)\ (\tilde{A}^{-1}\ B)^{\dagger}\ ]\tilde{A}^{-1}\ [(A + FL)x_{sp} + FHz_{\infty}]. \tag{23}$$

Another approach developed in [13] was to minimize the disturbance effects in (17). In this instance, $u_{ds}(t)$ and $u_{dw}(t)$ are designed to minimize

$$\left\| (A+FL)x_{sp} + Bu_{ds}(t) \right\| \quad \text{and} \quad \left\| FHz(t) + Bu_{dw}(t) \right\| , \tag{24}$$

respectively. It was shown that, for $Q \equiv I$, the minimum norm control components which minimize the norms in (24) are given by

$$u^*_{ds}(t) = -B^{\dagger}(A + FL)x_{sp} \tag{25}$$

$$u^*_{dw}(t) = -B^{\dagger}FHz(t). \tag{26}$$

Substitution of (25) and (26) into (17) results in the following expression for the error dynamics,

$$\dot{\varepsilon}(t) = \tilde{A}\varepsilon(t) - (I - BB^{\dagger})[(A + FL)x_{sp} + FHz(t)]. \tag{27}$$

Under the same assumptions as to the existance of a steady-state solution made with reference to (17), the unique steady-state solution for (27) is

$$\varepsilon^*_{ss} = \tilde{A}^{-1}\ (I - BB^{\dagger}\ )[(A + FL)x_{sp} + FHz_{\infty}]. \tag{28}$$

An example problem was worked-out in [13] using the controller pair given by (21) and (22) and the controller pair given by (25) and (26). A comparison of results indicated that use of the first controller pair did result in a smaller steady-state error. In [14], additional data was presented which indicated that, for the same conditions considered in [13], the first controller pair also resulted in less transient excursion in $\varepsilon(t)$.

IV. PLANT AND DISTURBANCE MODELS

The plant state and output models used for the state set-point regulator example of [13,14] and for examples to be presented in this report are

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u + \begin{pmatrix} 1 \\ 1 \end{pmatrix} w \tag{29}$$

$$y = (1,\ 0)\ x \tag{30}$$

The external disturbance model is

$$w = z \tag{31}$$

$$z = \sigma(t)\ , \tag{32}$$

i.e., the external disturbance was considered to be constant between σ arrivals. The target state set-point vector is given as

$$x_{sp} = (x_{sp,1},\ 0.)\quad . \tag{33}$$

The plant given by (29) and (30) is completely controllable. For the purpose of the examples, it was assumed in [13,14], and will be assumed in this report also, that all necessary state information is available from an ideal reconstructor.

V. GEOMETRICAL CONSIDERATIONS

Since the control distribution matrix B of (29) is a 2x1 matrix of rank 1, it does not span the state space, which is two-dimensional in this example. Hence, $Bu_d(t)$ will have a limited set of attainable points in the state space. Also, the external disturbance distribution matrix F is of rank 1 and thus, Fw(t) will have a limited range of action in the state space. As can be seen in Figure 1, the lines of action of Bu_d and Fw are not colinear. Furthermore, as shown in Figure 2, the line of action of the set-point disturbance term, in the error state space, is not colinear with the line of action of the control. Hence, no u_d exists which will completely absorb a non-zero external disturbance or a set-point disturbance resulting from a non-zero target state set-point.

Given that this situation exists and that a design objective is to minimize the effects of the uncancellable disturbance, one thus attempts to design u_d so as to achieve this objective in some fashion. To illustrate the action of the controllers shown in Section III, consider the following. With respect to the vectors Fw_1 and Bu_{d1} shown on Figure 1, one approach to the minimization problem is to first express the vector Fw as the sum of two component vectors, one lying in the column range space of B, R(B), and one lying in the orthogonal complement to the column range space of B, $R(B)^{\perp}$. This makes it easy to see that the component lying in $R(B)^{\perp}$, which is the component that is uncancellable, is minimized if the component lying in R(B) is the orthogonal projection of Fw_1, onto R(B). In essence, this result is provided by the controllers shown in Section III. For instance, if u_{d1} is chosen as

$$u_{d1} = -B^{+} Fw_1 \tag{34}$$

then

$$Bu_{d1} + Fw_1 = -BB^{+} Fw_1 + Fw_1 = (I-BB^{+})Fw_1 \quad , \tag{35}$$

where $(I-BB^{+})$ is the projector of Fw_1 on $R(B)^{\perp}$ along R(B) and BB^{+} is the projector of Fw_1 on R(B) along $R(B)^{\perp}$. The uncancellable part of the external disturbance is thus the component in $R(B)^{\perp}$.

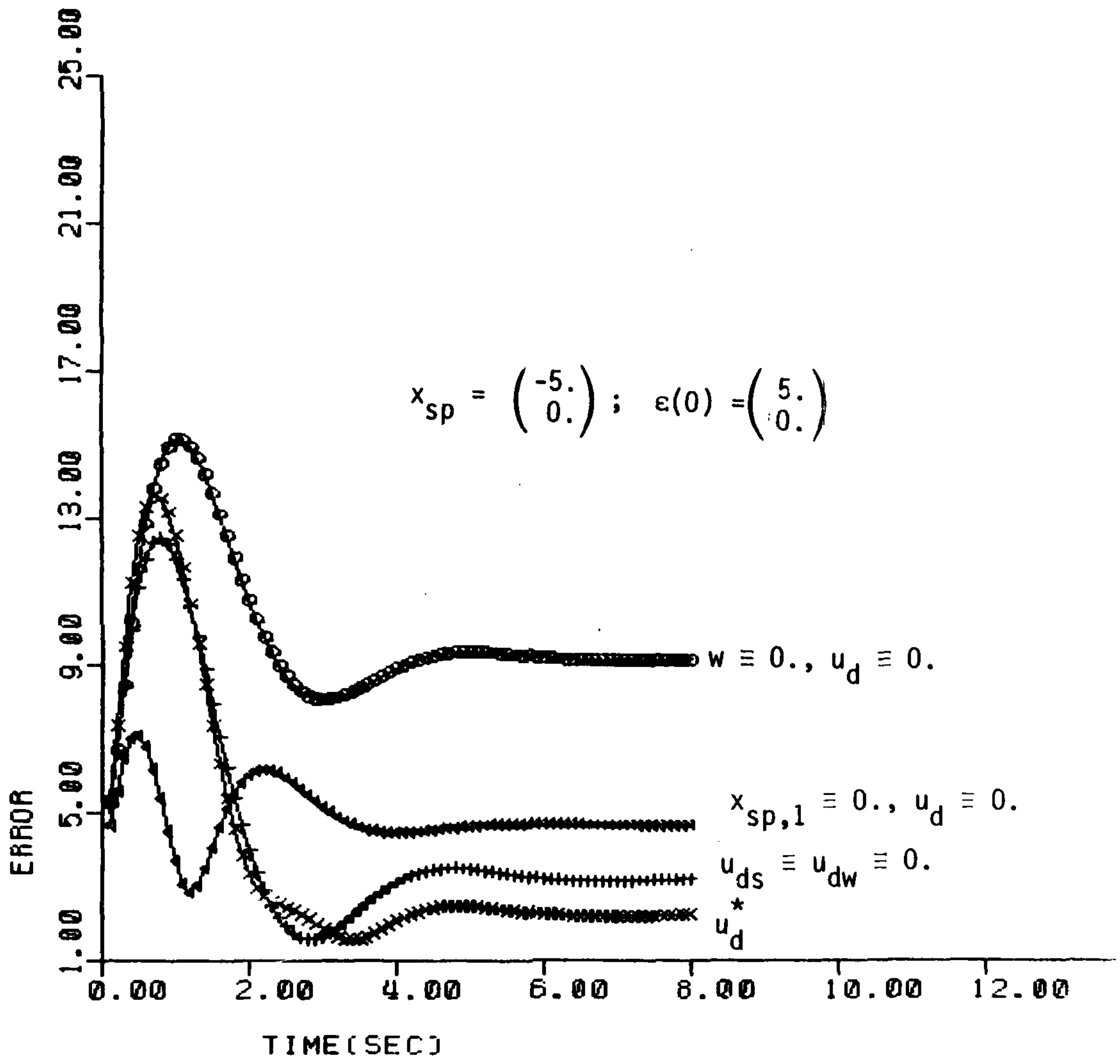

Figure 12. Plant responses to different disturbance and disturbance control conditions, controller pair (21) & (22).

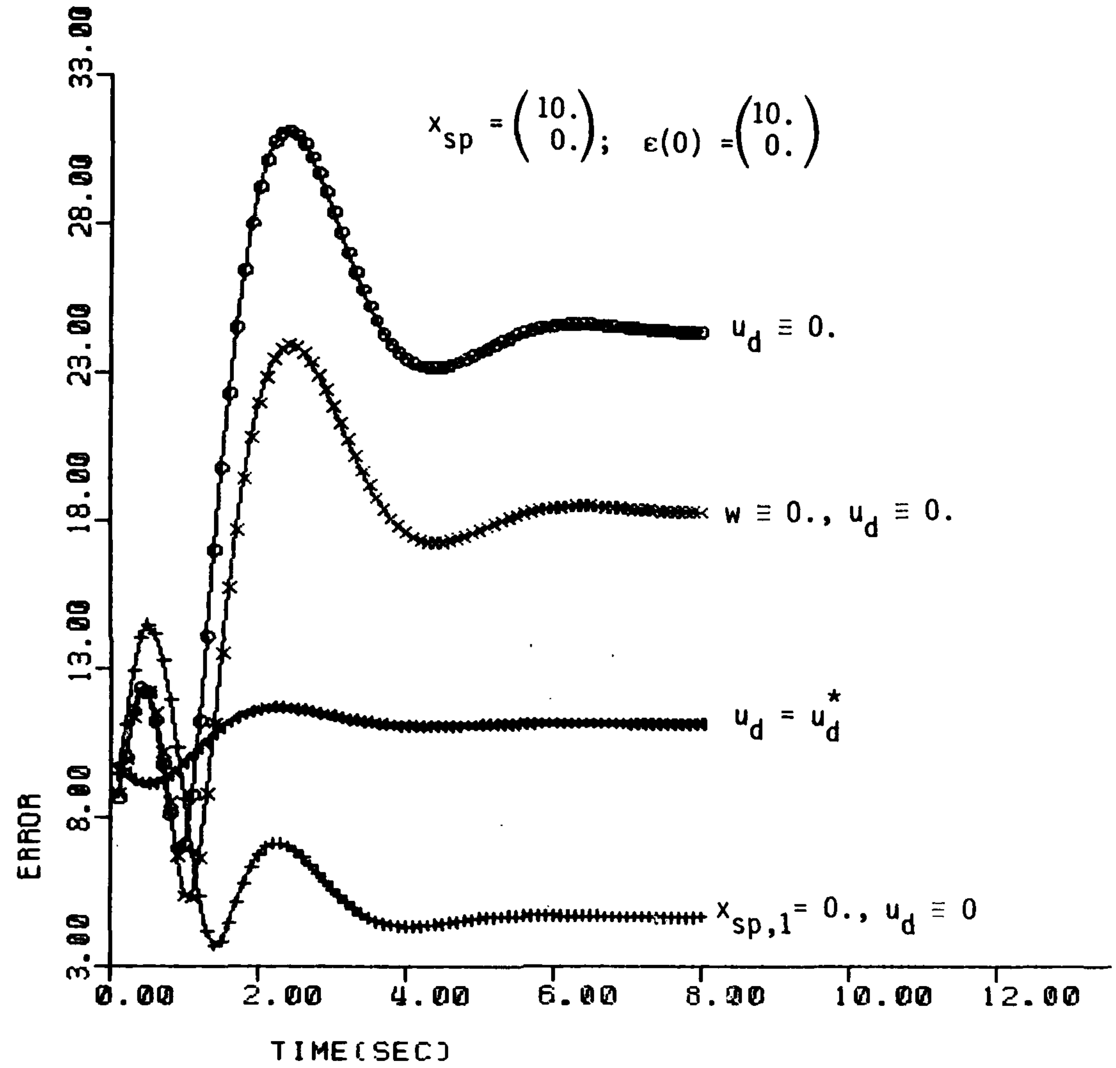

Figure 11. Plant responses to different disturbance and disturbance control conditions, controller pair (21) & (22).

VII. EXTERNAL DISTURBANCE UTILITY

A. Introduction

In [13], the "utility" U of an external disturbance w was defined as

$$U = \left\| \varepsilon_{ss} \right\|_{w = 0} - \left\| \varepsilon_{ss} \right\|_{w \neq 0}. \quad (41)$$

If U is positive, then w has aided in reducing the final error. If U is negative, then w has further increased the error. Note that, if the target set-point is the origin, such that the set-point disturbance is zero, a non-zero, uncancellable external disturbance will always exhibit a negative utility. It was also shown in [13] that two conditions must be satisfied in order for an external disturbance to have a positive utility. In order for w to provide a positive utility it must satisfy a magnitude condition given by

$$\sum_{i=1}^{n} (\bar{a}^i + \bar{f}^i)^2 - \sum_{i=1}^{n} (\bar{a}^i)^2 < 0 \quad (42)$$

and an angle condition given by

$$\langle 2\bar{a} + \bar{f}, \bar{f} \rangle < 0., \quad (43)$$

i.e.,

$$90° < \theta < 270°, \quad (44)$$

where

$\bar{a}$ is the component of Ax_{sp} lying in $R(B)^{\perp}$,

$\bar{f}$ is the component of Fw lying in $R(B)^{\perp}$,

θ is the angle between the vectors $2\bar{a}+\bar{f}$ and $\bar{f}$, and

$\langle \cdot \rangle$ denotes the inner product.

It is interesting to consider the results obtained for the cases of Figures 3 through 6 when the disturbances are alternately set to zero and the disturbance minimization control u_d is set to zero. Figures 11 to 14 present the results obtained when this is done. Consider first the case where u_d is designed to minimize ε_{ss}. The separate effects due to the external and set-point disturbances, with no disturbance control active, for $x_{sp} = (10.,0.)^T$ are shown in Figure 11. Note that when both disturbances are input, again with no disturbance control active, the resultant error is additive. In Figure 12 however, where $x_{sp} = (-5.,0.)^T$, under the same conditions the error when both disturbances are present is less than that due to the set-point disturbance alone. In this latter instance, the external disturbance has aided in reducing the error, i.e., has exhibited a positive utility. Next,

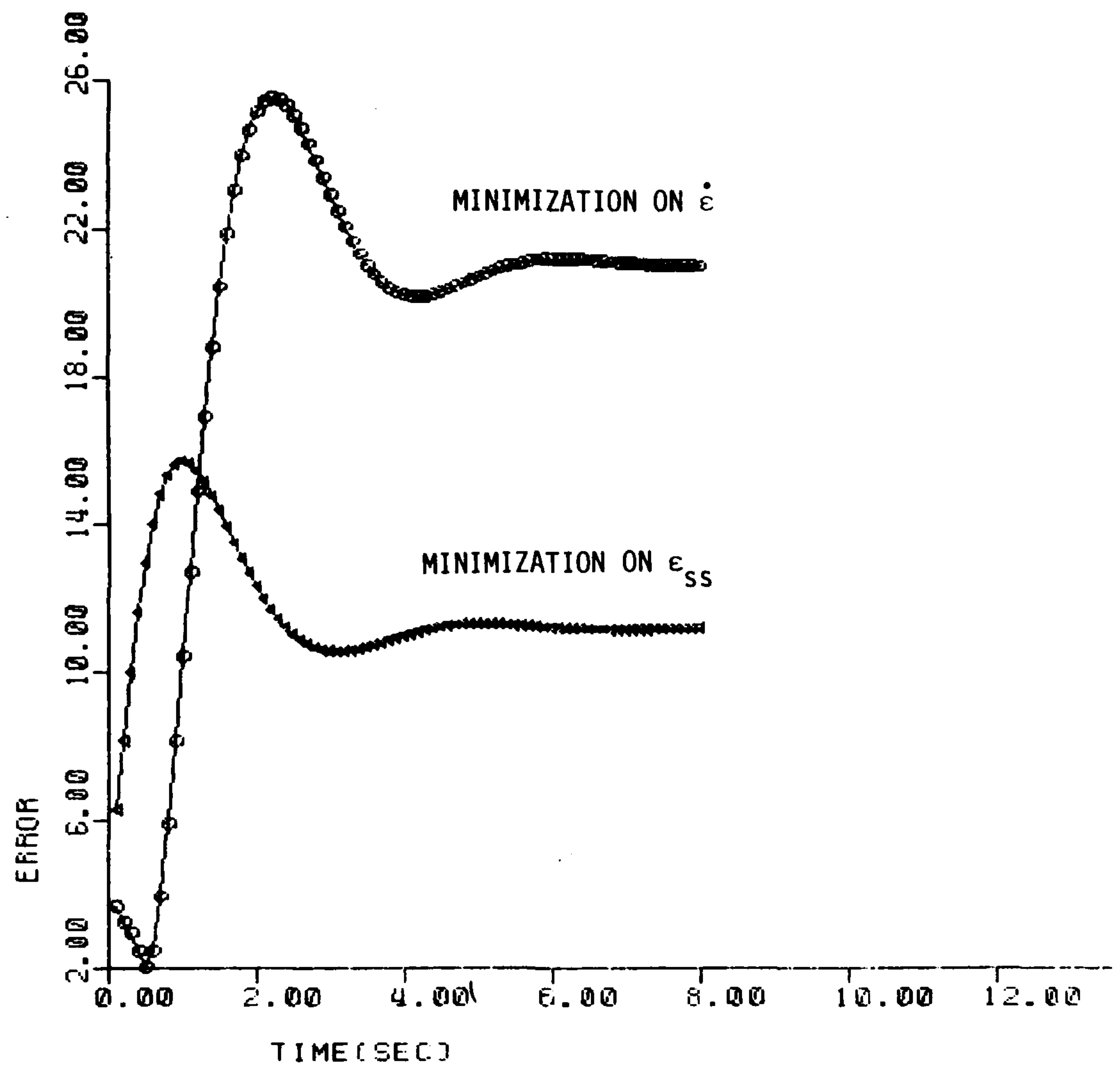

Figure 10. Plant response with $x_{sp} = (-15.,0.)^T$ and $\varepsilon(0) = (-5.,0.)^T$, w = 5.

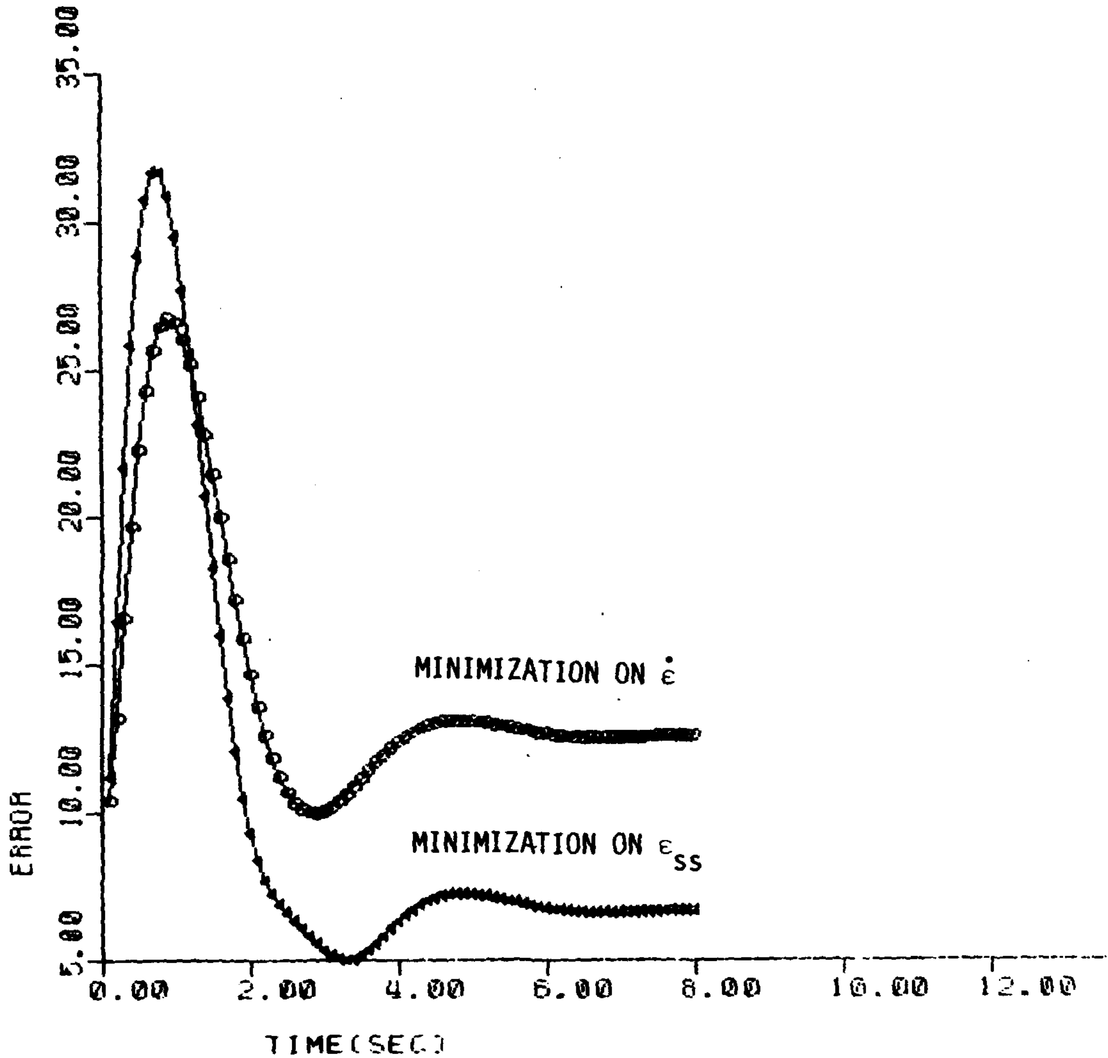

Figure 9. Plant response with $x_{sp} = (-15.,0.)^T$ and $\varepsilon(0) = (15.,0.)^T$, $w = 5.$

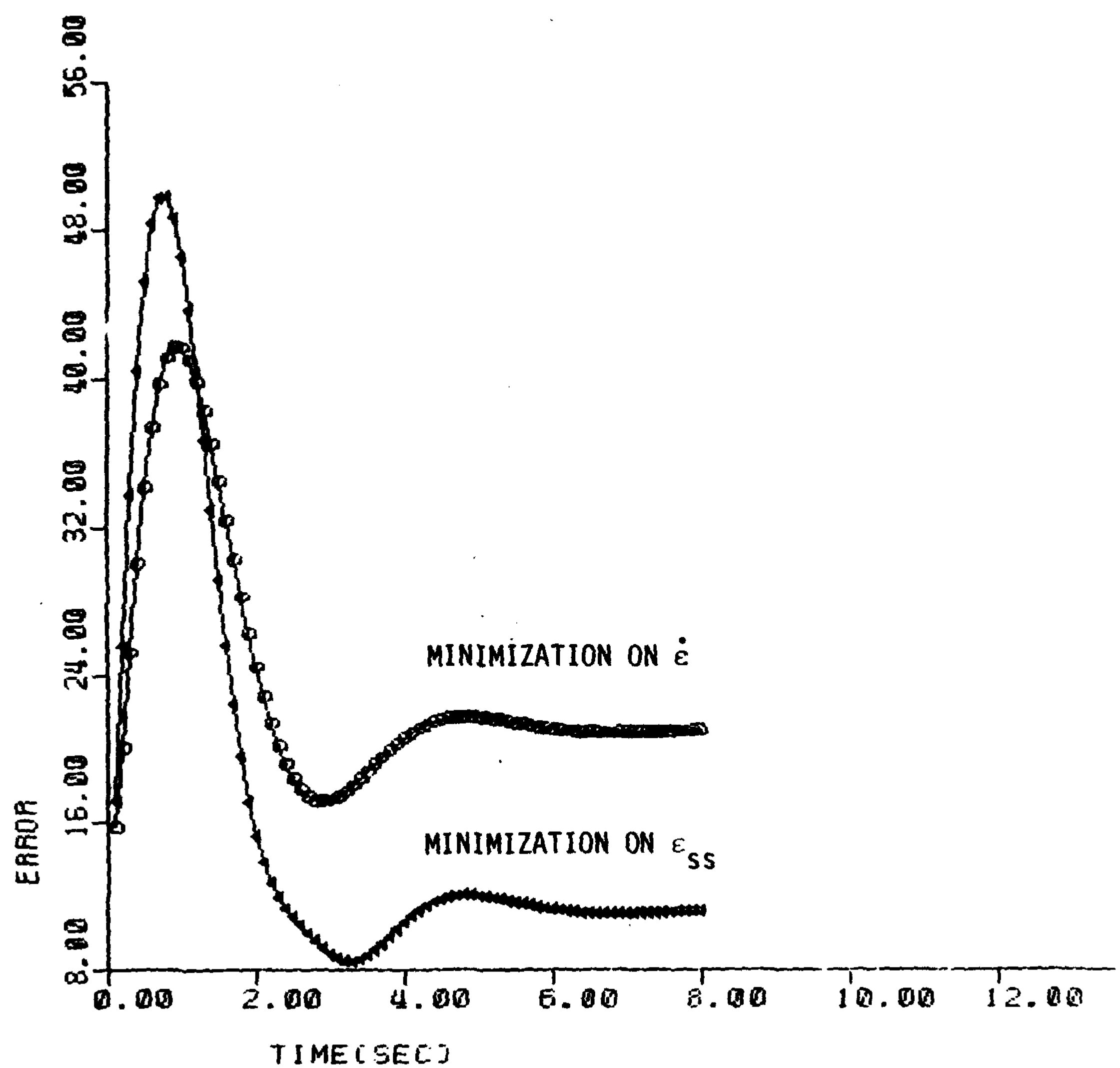

Figure 8. Plant response with $x_{sp} = (5.,0.)^T$ and $\varepsilon(0) = (-10.,0.)^T$, $w = 5.$

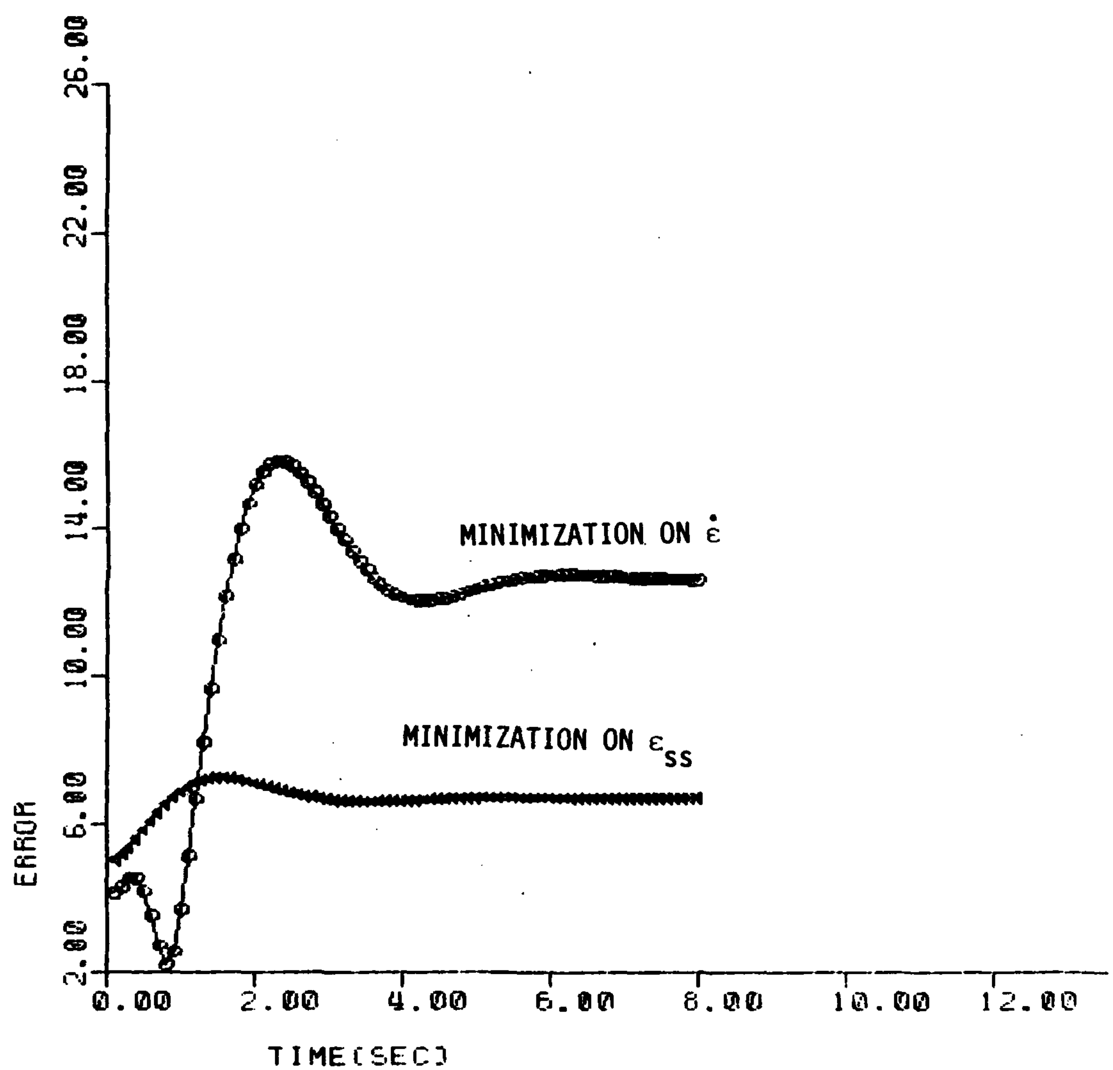

Figure 7. Plant response with $x_{sp} = (5.,0.)^T$ and $\varepsilon(0) = (5.,0.)^T$, $w = 5.$

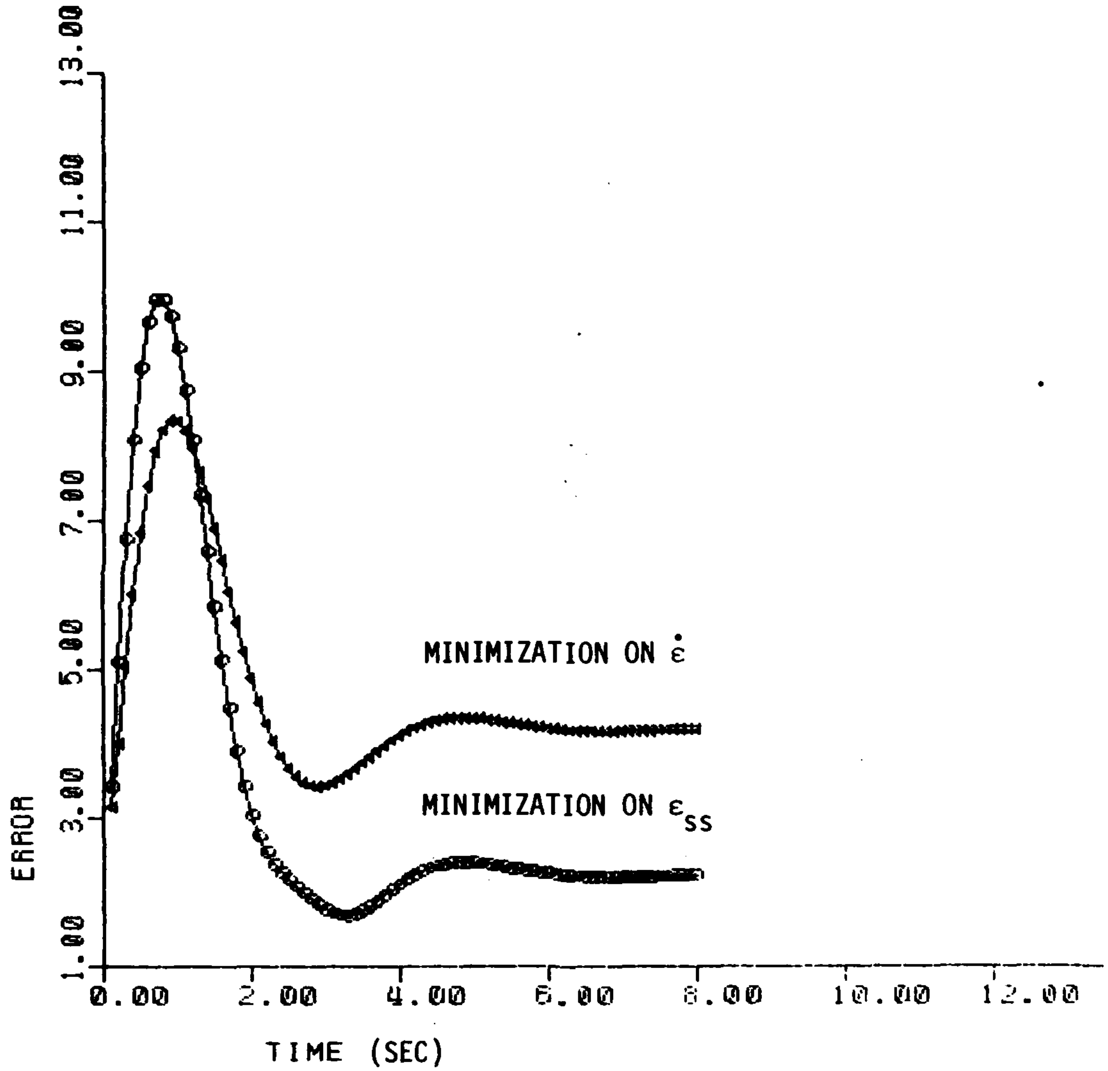

Figure 6. Plant response with $x_{sp} = (-5.,0.)^T$ and $\varepsilon(0) = (3.,0.)^T$, w = 5.

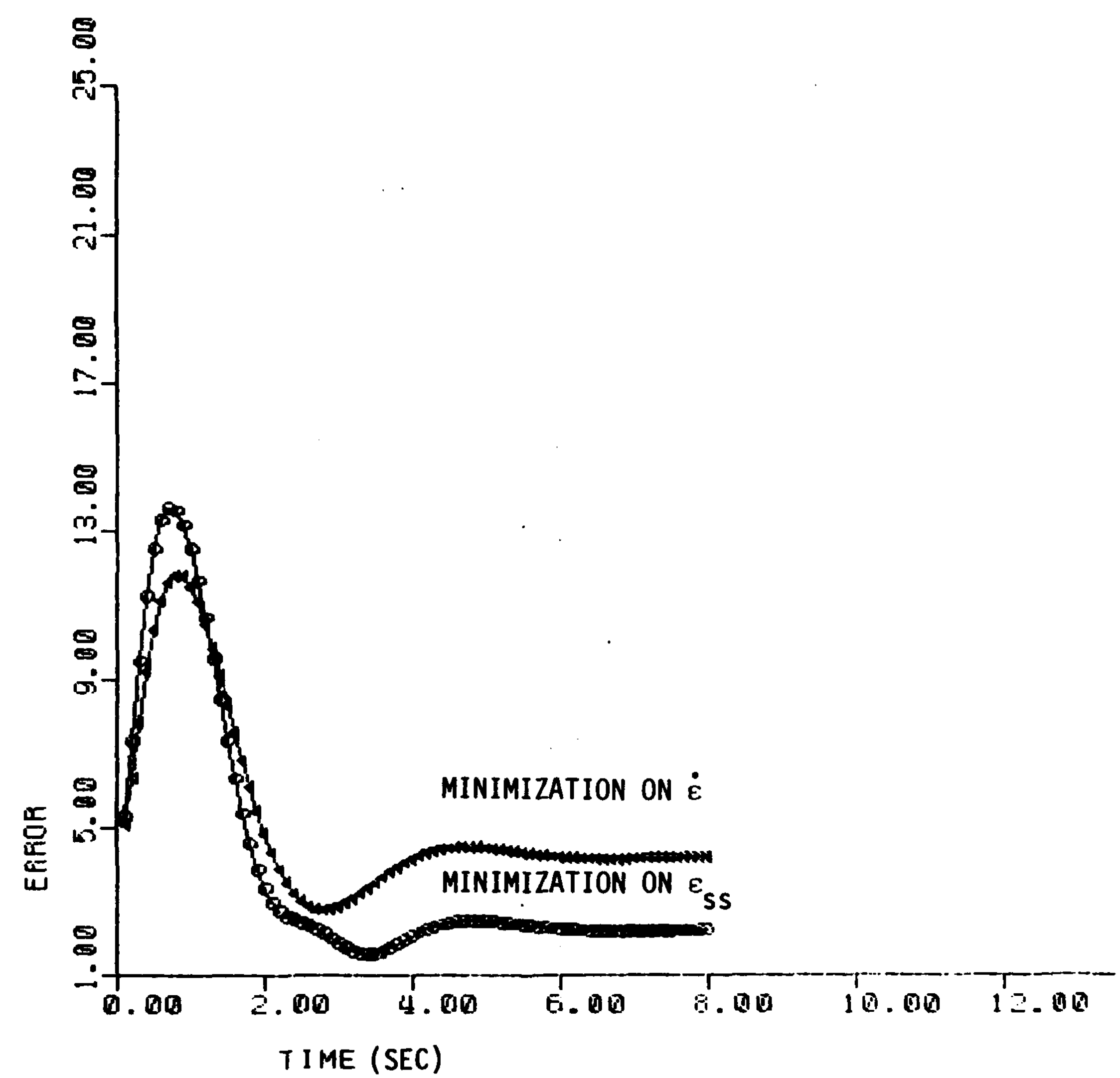

Figure 5. Plant response with $x_{sp} = (-5.,0.)^T$ and $\varepsilon(0) = (5.,0.)^T$, $w = 5$.

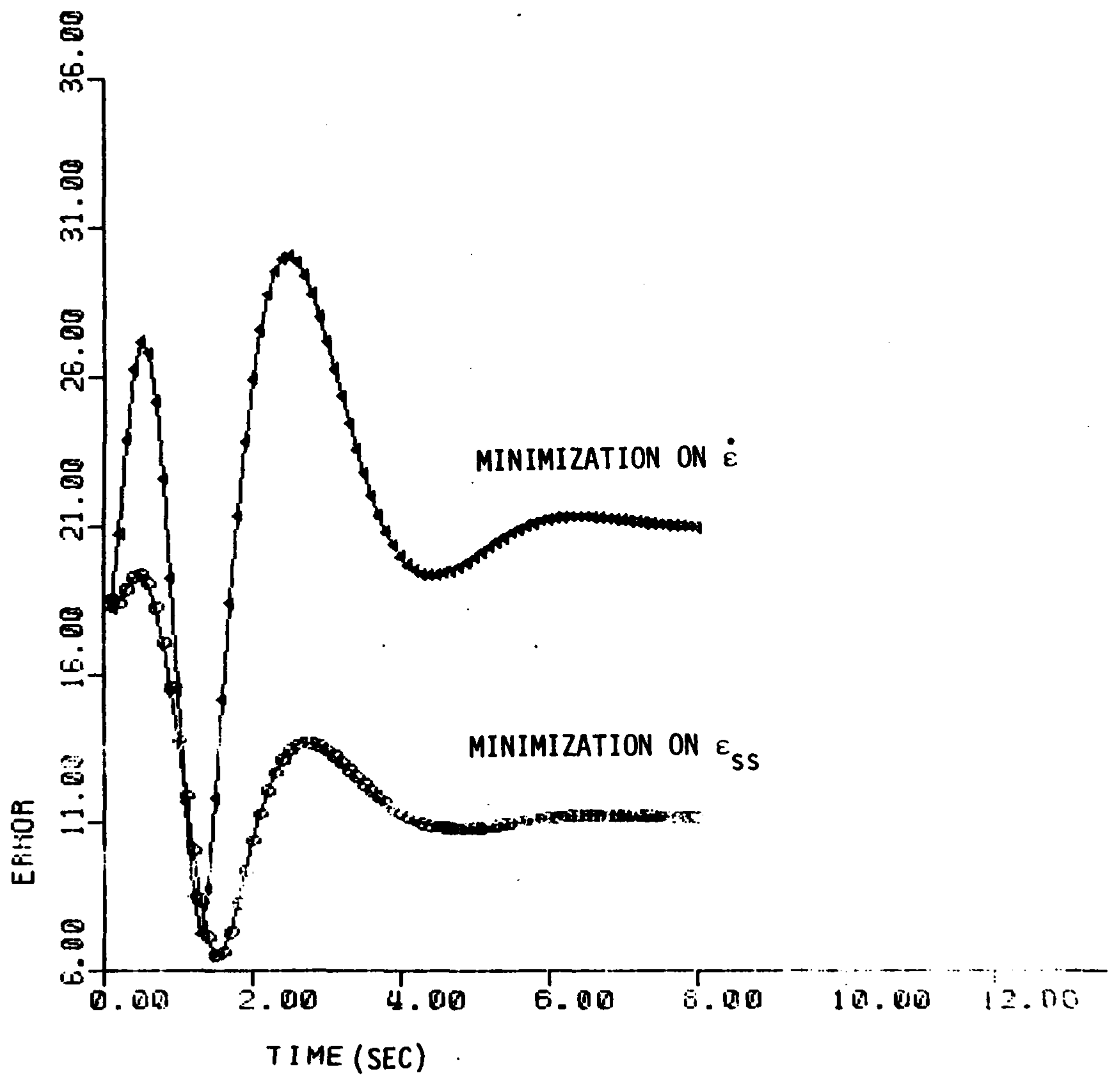

Figure 4. Plant response with $x_{sp} = (10.,0.)^T$ and $\varepsilon(0) = (20.,0.)^T$, $w = 5$.

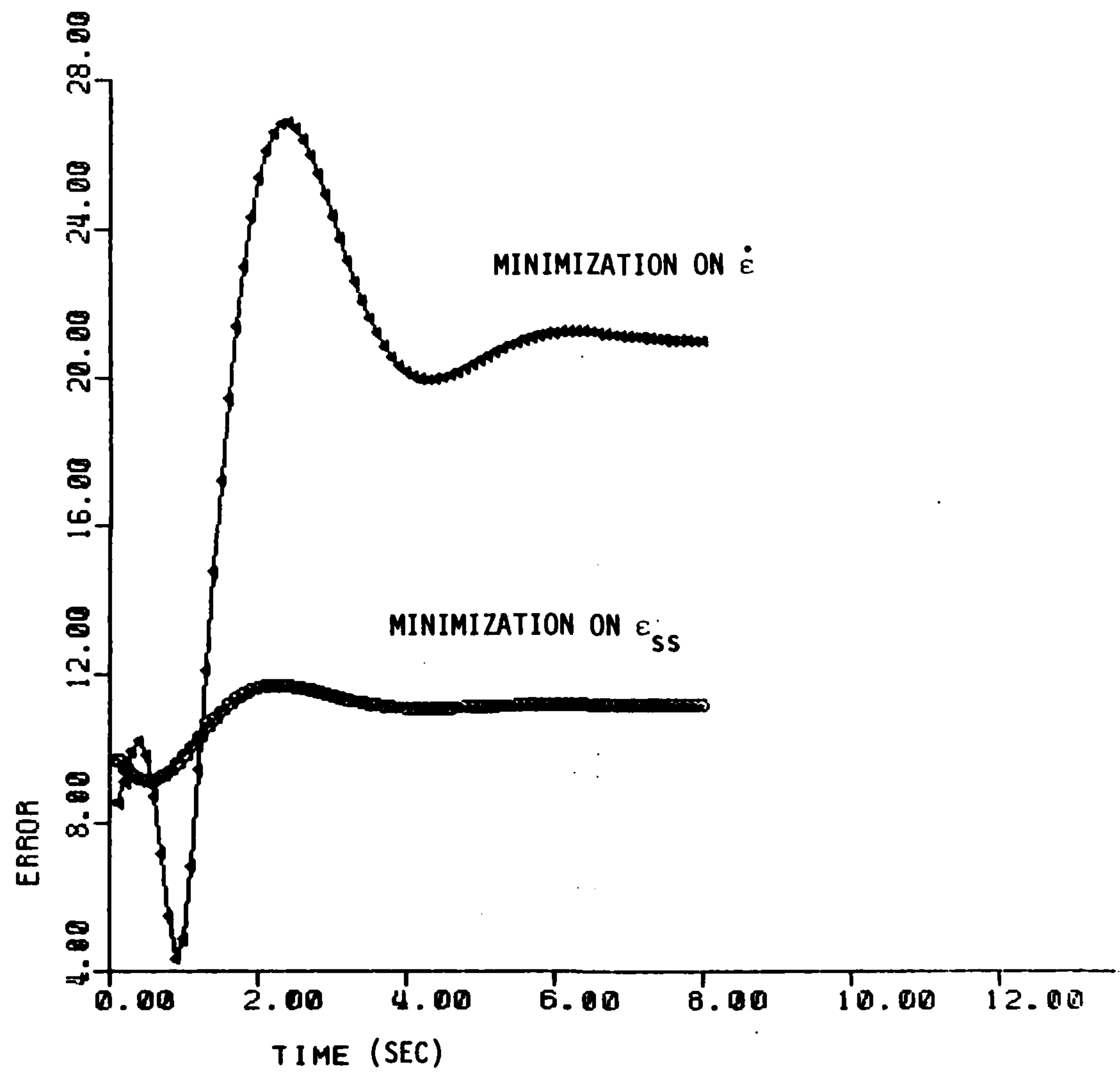

Figure 3. Plant response with $x_{sp} = (10.,0.)^T$ and $\varepsilon(0) = (10.,0.)^T$, $w = 5$.

VI. PERFORMANCE COMPARISON WITH CONSTANT DISTURBANCE

As was mentioned in Section III,the plant and disturbance models given in Section IV were used to work out ([13,14]) an example problem in order to illustrate the performance resulting from application of the controller pairs given by Equations (21, 22) and (25, 26), i.e., minimizing the effects of the disturbance on the steady-state error versus minimizing the effects of the disturbance in the differential equation describing the error.

In [13], it was shown that the controller given by Equations (21) and (22) are

$$u_{ds}^{*} = -2.3447x_{sp,1} \tag{36}$$

$$u_{dw}^{*} = -1.6723z, \tag{37}$$

and the controllers given by Equations (25) and (26) are

$$u_{ds}^{*} = -0.2x_{sp,1} \tag{38}$$

$$u_{dw}^{*} = -0.6z. \tag{39}$$

The stabilization control u_p, designed according to Equations (15) and (16), was chosen to be

$$u_p(t) = -K\varepsilon(t) = (3., 0.36)\varepsilon(t). \tag{40}$$

In [14], results were presented which show the transient behavior of the error for each pair of controllers for a case with $\varepsilon(0) = (10.,0.)^T$. Figure 3 presents these results. As can be seen from Figure 3, for the conditions assumed, designing u_d to mimimize the steady-state error resulted in the smaller steady-state error and also in a better transient excursion performance.

It is of interest to know if the controllers designed as in Equations (36) and (37) will always result in the best overall performance. This controller design technique will always result in the smallest steady-state error which it is possible to achieve for a given set of conditions, i.e, a given target set-point, initial conditions and constant external disturbance, since it results in controllers which are designed specifically to minimize the steady-state error. But what about the transient excursion performance?

In order to examine this question, several additional cases have been simulated. Results are presented in Figures 4 to 10. In all cases, the controller pair designed to minimize ε_{ss} did give the smallest steady-state error. Neither controller pair exhibited a consistent advantage in transient excursion performance; however, the performance associated with controller pair (36, 37) was, in all cases, at least as good as that associated with controller pair (38, 39).

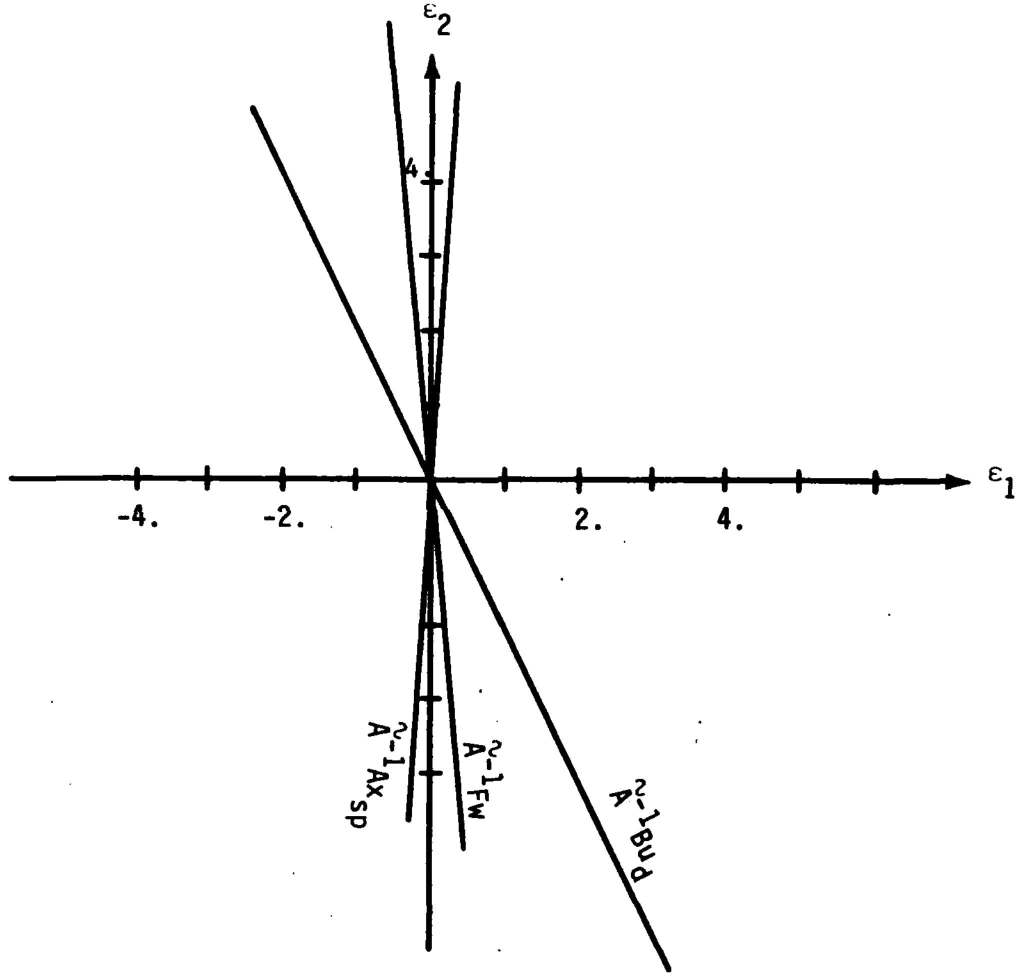

Figure 2. Line of action for disturbances and control in error state space.

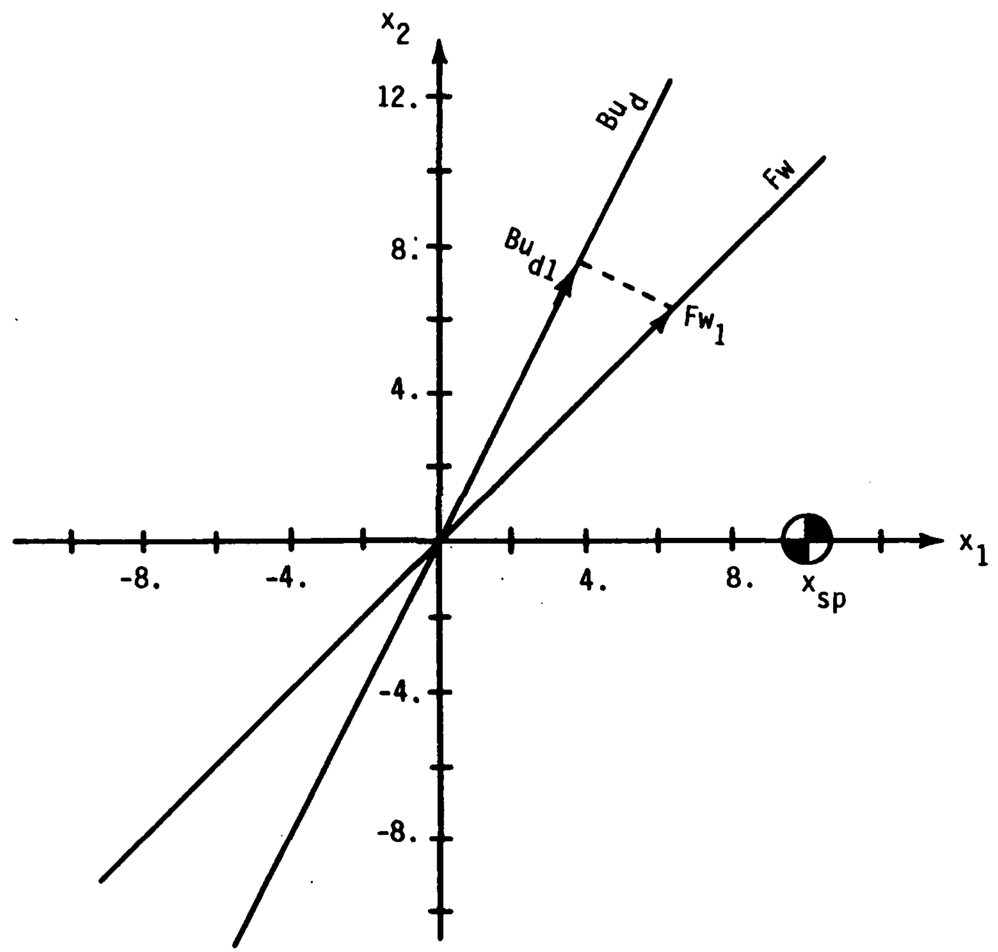

Figure 1. Line of action of external disturbance and control in plant state space.

consider the case where u_d is designed to minimize the disturbance effects on ε. The results for this case are shown in Figures 13 and 14. Again, it can be seen that for $x_{sp} = (-5.,0.)^T$ the external disturbance assists in reducing the total error.

This Section contains two examples which show the region of positive utility of external disturbances, the geometry involved in the state space and several other interesting results for the set-point regulator model of Section IV. The first example is for the case with $x_{sp} = (10.,0.)^T$ and the second is for the case with $x_{sp} = (-5.,0.)^T$.

B. Example 1

For this first example, the target state set-point is $x_{sp} = (10.,0.)^T$. The set-point disturbance vector is thus given by

$$Ax_{sp} = \begin{bmatrix} 1. & 1. \\ 0. & 1. \end{bmatrix} \begin{pmatrix} 10. \\ 0. \end{pmatrix} = \begin{pmatrix} 10. \\ 0. \end{pmatrix} . \tag{45}$$

The external disturbance vector in state space is given by

$$Fw = FHz = \begin{pmatrix} 1. \\ 1. \end{pmatrix} (1.\ )(5.\ ) = \begin{pmatrix} 5. \\ 5. \end{pmatrix} . \tag{46}$$

Figure 15 is a plot of the state space showing the two disturbance vectors and the line of action, $R(B)$, of the control.

As was mentioned in Section V, each of the disturbance vectors can be expressed as the sum of two component vectors [13], one lying in $R(B)$ and one lying in $R(B)^{\perp}$. The component of each vector lying in $R(B)$ can be found by multiplying the vector by $BB^{\dagger}$ and the component lying in $R(B)^{\perp}$ can be found by multiplying the vector by $(I-BB^{\dagger})$. The components in $R(B)^{\perp}$ are then used in checking for satisfaction of conditions (42) and (44). Instead of (42), one can also use the inequality [13],

$$\left\| \bar{f} \right\| < \left\| 2\bar{a} \right\| \tag{47}$$

to establish an upper bound on the magnitude which $\bar{f}$ can have and still allow w to exhibit a positive utility.

For the set-point disturbance of (45), the component lying in $R(B)^{\perp}$ is found to be

$$\bar{a} = (I - BB^{\dagger})\, Ax_{sp} = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 0.2 \end{bmatrix} \begin{pmatrix} 10. \\ 0. \end{pmatrix} = \begin{pmatrix} 8. \\ -4. \end{pmatrix} \tag{48}$$

and the component of (46) lying in $R(B)^{\perp}$ is

$$\bar{f} = (I - BB^{\dagger})FHz = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 0.2 \end{bmatrix} \begin{pmatrix} 5. \\ 5. \end{pmatrix} = \begin{pmatrix} 2. \\ -1. \end{pmatrix} . \tag{49}$$

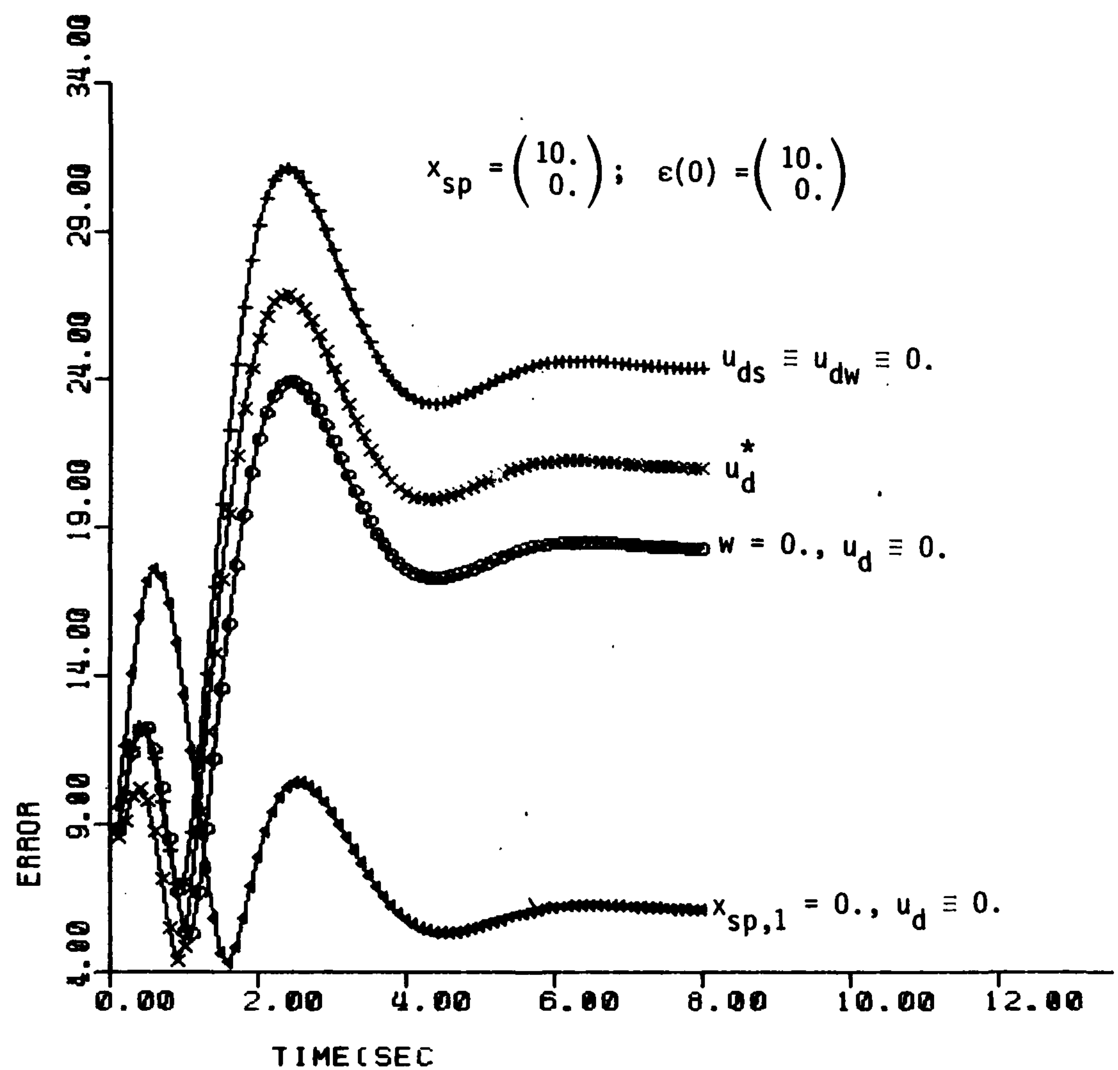

Figure 13. Plant responses to different disturbance and disturbance control conditions, controller pair (25) & (26).

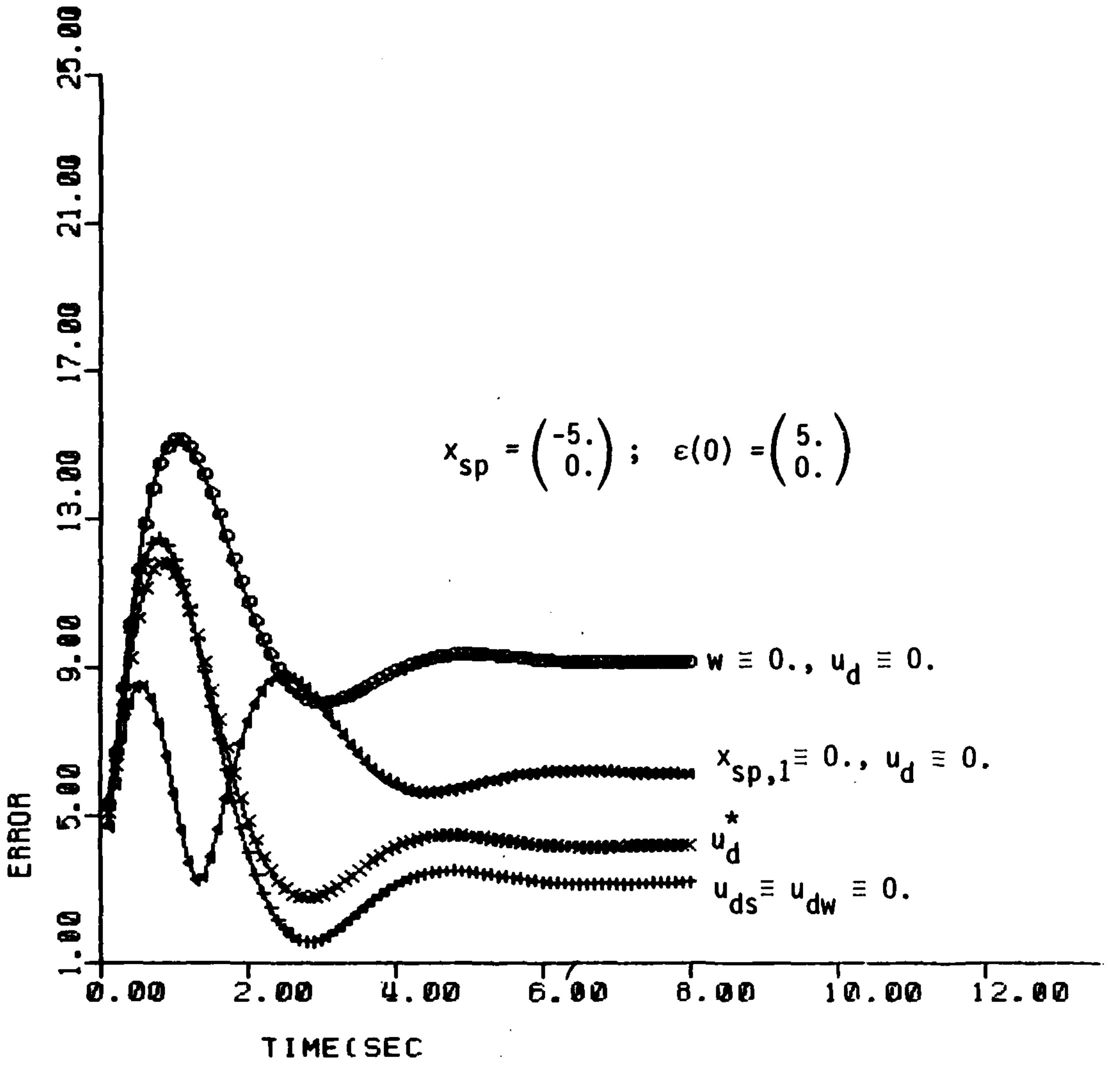

Figure 14. Plant responses to different disturbance and disturbance control conditions, controller pair (25) & (26).

These components are also shown in Figure 15. As can be seen, they act in the same direction and hence will reinforce each other, i.e., one would expect w to exhibit a negative utility in this case. Substituting from (48) and (49) into (42), one obtains

$$\left\| \bar{a} + \bar{f} \right\|^2 - \left\| \bar{a} \right\|^2 = 125. - 80. > 0. \tag{50}$$

and it is evident that $\Theta = 0°$; therefore, w does not satisfy the conditions for U>0.

According to the requirement given by (44), for a positive utility in this example $\bar{f}$ must lie to the "left" of R(B). Since the line of action of w is a line with a slope of +1 in the state space, those external disturbances which may exhibit a positive utility must be negative. Furthermore, the bound on the allowable magnitude of $\bar{f}$, as indicated by (47), specified that

$$\left\| \bar{f} \right\| < \left\| 2\bar{a} \right\| = 17.89 . \tag{51}$$

Since $\bar{f} = (I-BB^{r})FHz$, one has that

$$\begin{pmatrix} \bar{f}_1 \\ \bar{f}_2 \end{pmatrix} = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 0.2 \end{bmatrix} \begin{pmatrix} z \\ z \end{pmatrix} = \begin{pmatrix} 0.4z \\ -0.2z \end{pmatrix} \tag{52}$$

so that

$$\left\| \bar{f} \right\| = \sqrt{0.2z^2} < 17.89, \tag{53}$$

therefore,

$$\left\| z \right\| < 40. \tag{54}$$

For w to exhibit a positive utility when $x_{sp} = (10.,0.)^T$, FHz must be negative, with the magnitude of z less than 40. Figure 16 indicates the regions of positive and negative utility for possible external disturbance magnitudes. Since FHz is a line in state-space, the utility regions of w are as follows:

$$\left.\begin{array}{r} w > 0. \\ w = 0. \\ -40. < w < 0. \\ w < -40. \end{array}\right\} \begin{array}{l} U < 0. \\ U = 0. \\ U > 0. \\ U < 0. \end{array} \tag{55}$$

Figure 17 shows ε_{ss} as a function of external disturbance, with and without disturbance minimization control and with $u_d = 0$. Figure 18 is an expanded scale plot of the region in Figure 17 from w = -20. to w = -10. Figure 19 shows the percent reduction in ε_{ss} achieved by the two disturbance minimization controllers and Figure 20 is an expanded plot of a portion of Figure 19 for w between -20. and -10. Note from Figure 18 that there is a range of values for w for which the controller designed to minimize disturbance effects on $\dot{\varepsilon}$ causes an increase in ε_{ss} from what would be obtained if $u_d \equiv 0$. Also, note that there is one value for w, with u_d=0., which equals the performance of the controller designed to minimize ε_{ss}.

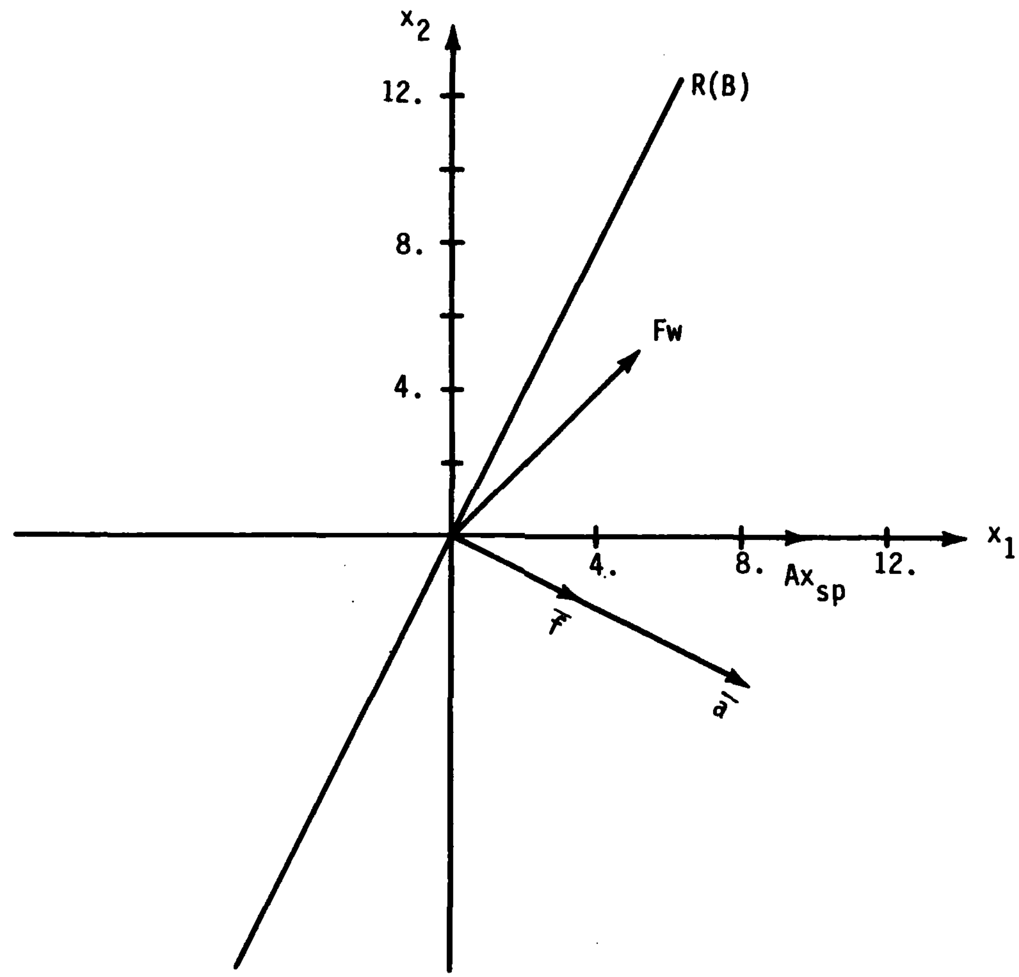

Figure 15. Geometry of disturbances relative to the line of action of the control, $x_{sp} = (10.,0.)^T$.

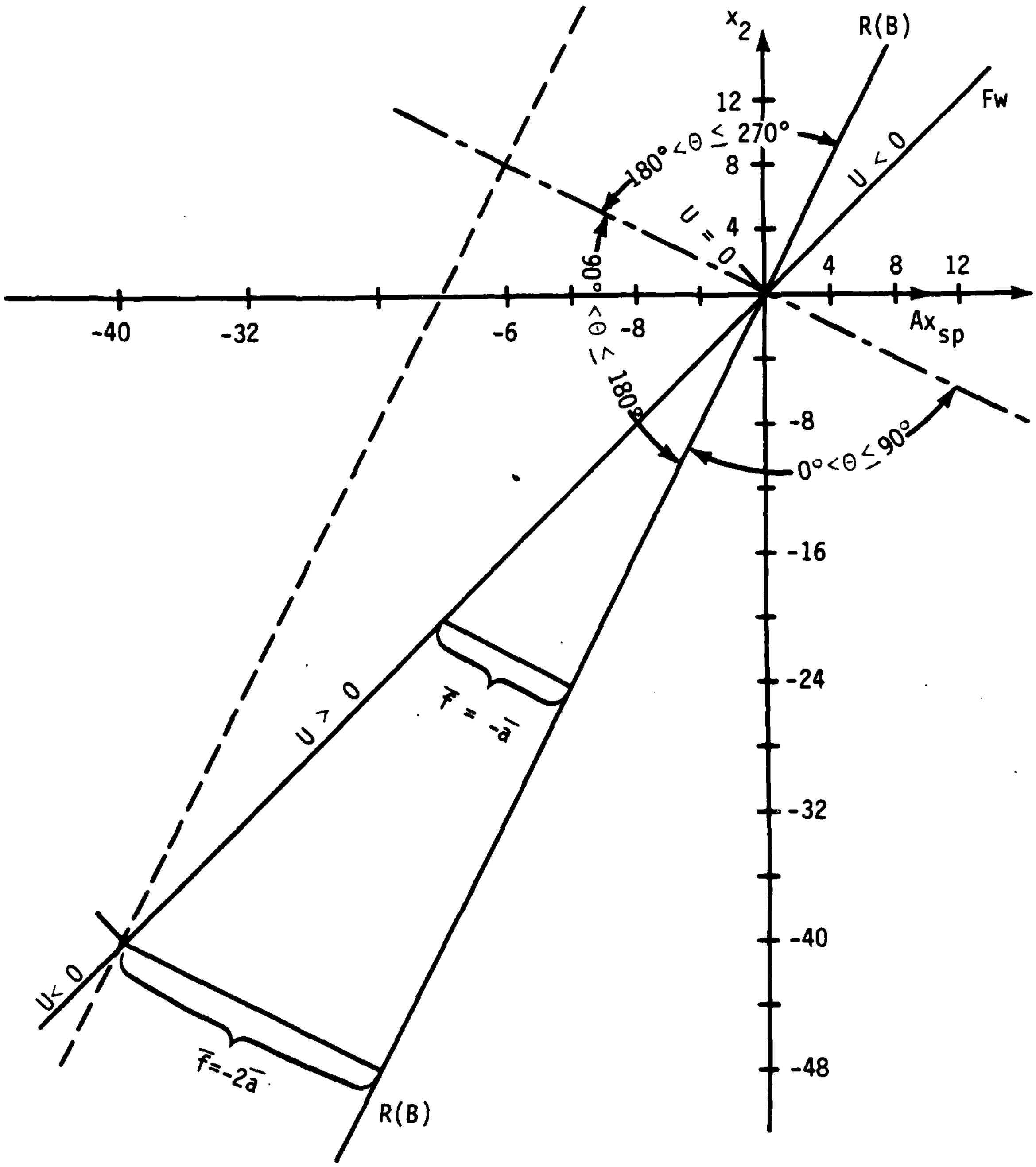

Figure 16. Region of positive utility for case with $x_{sp} = (10.,0.)^T$.

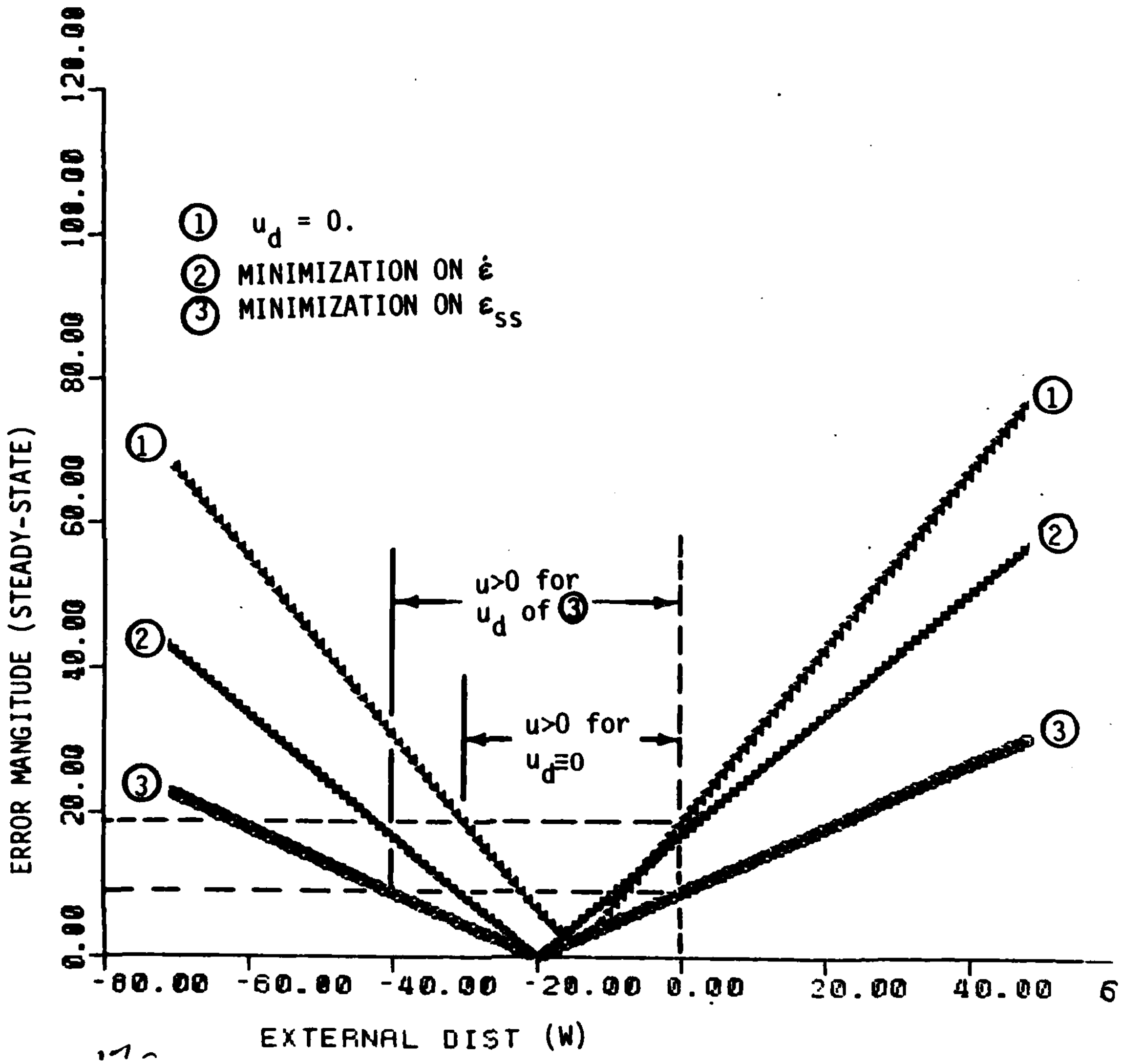

Figure 17. Steady-state error as a function of external disturbance magnitude, with $x_{sp} = (10.,0.)^T$.

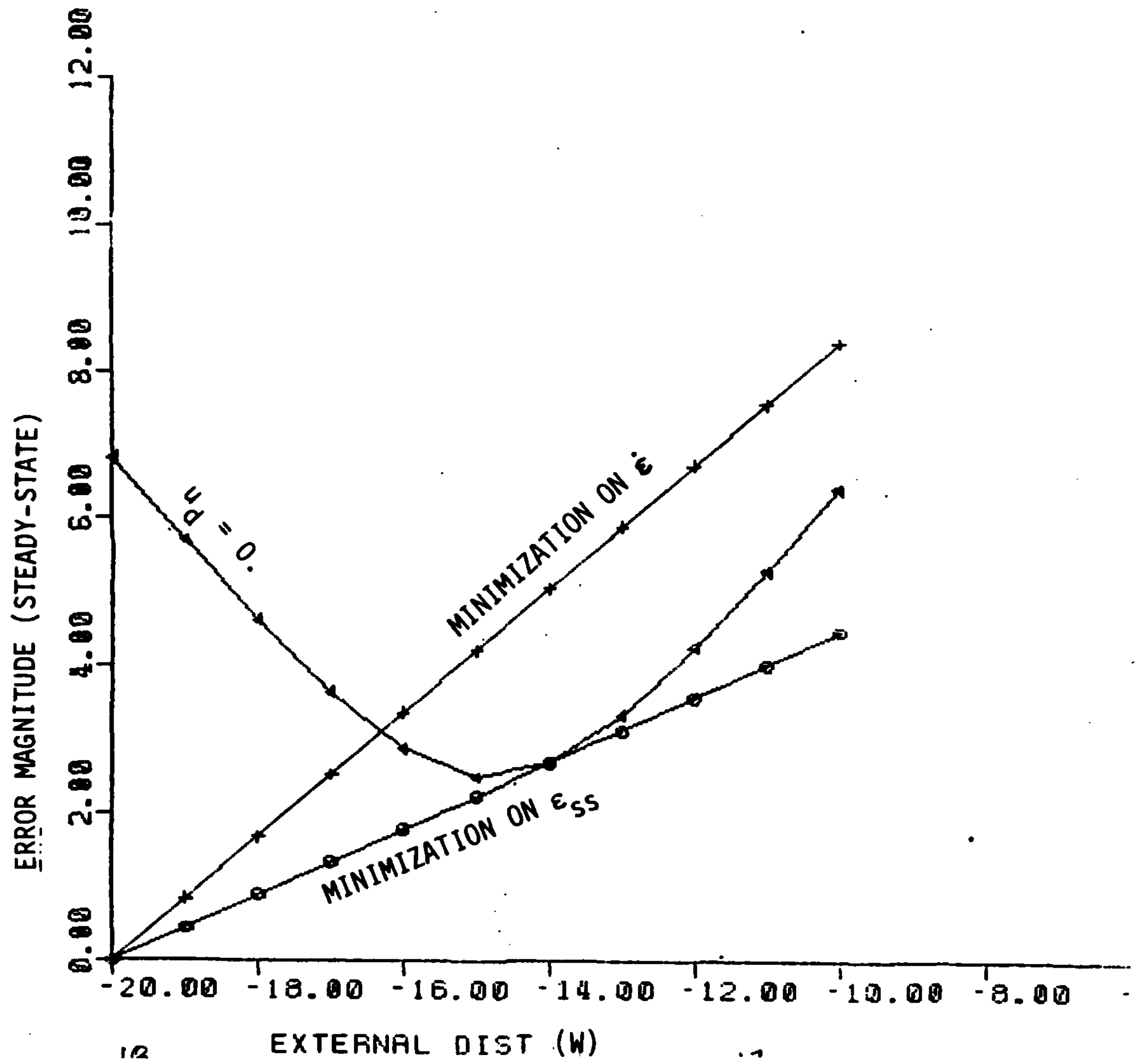

Figure 18. Expanded scale plot of a portion of Figure 17.

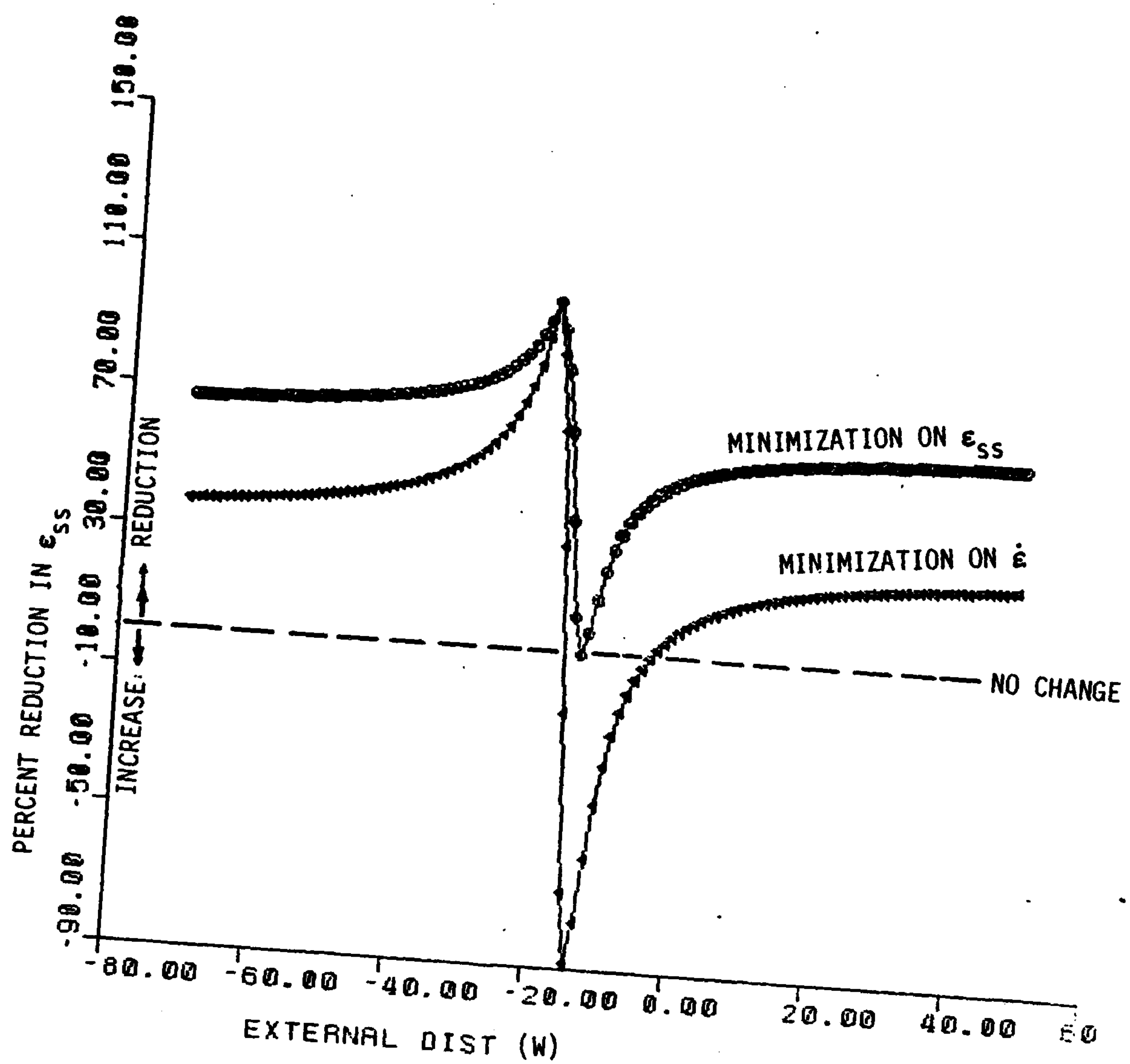

Figure 19. Percent reduction in steady-state error achieved by disturbance minimization controllers as compared to a case with $u_d = 0.$, $x_{sp} = (10.,0.)^T$.

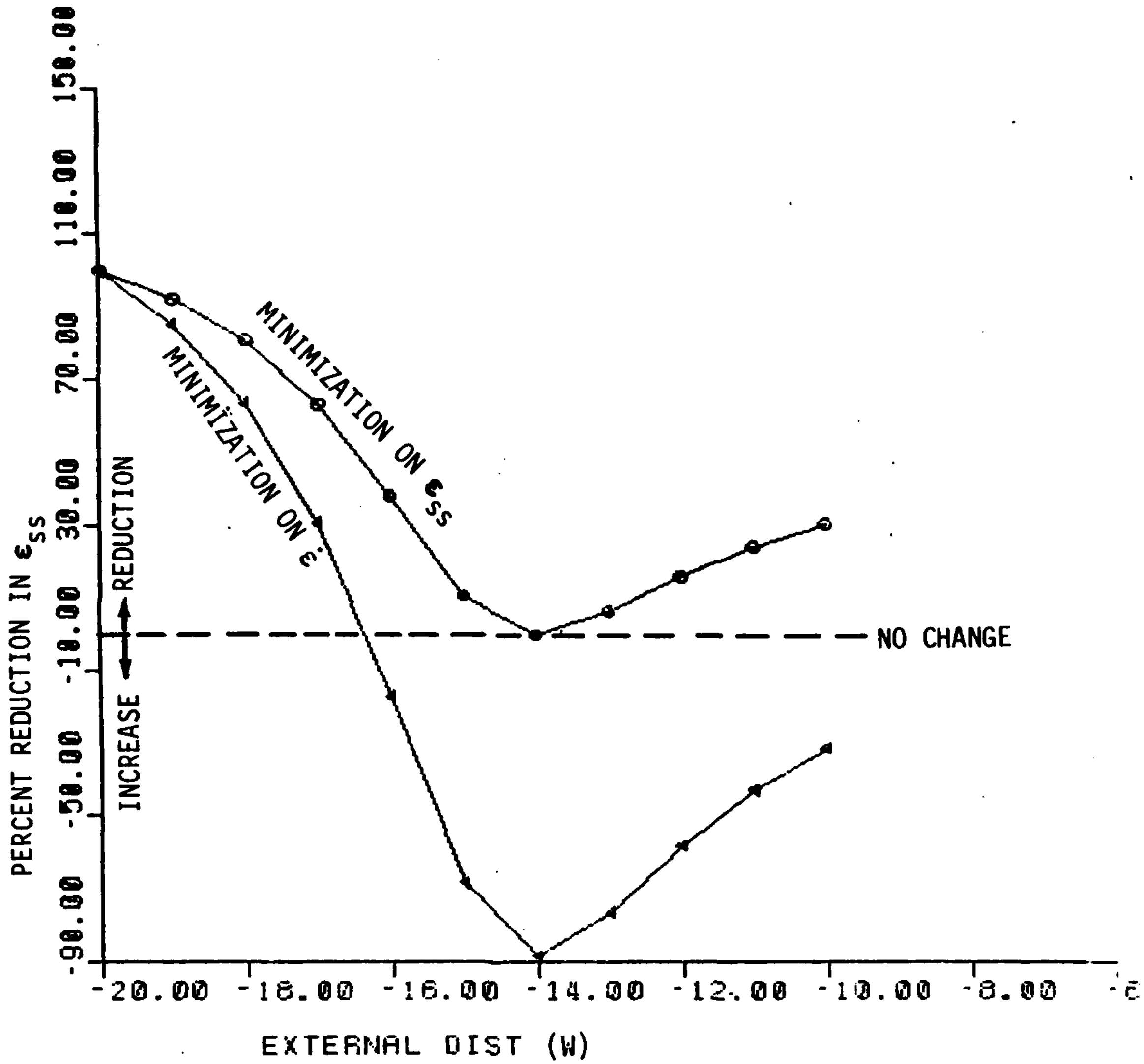

Figure 20. Expanded scale plot of a portion of Figure 19.

C. Example 2

In this example, the target state set-point is $x_{sp} = (-5.,0.)^T$. The set-point vector is thus given by

$$Ax_{sp} = \begin{bmatrix} 1. & 1. \\ 0. & 1. \end{bmatrix} \begin{pmatrix} -5. \\ 0. \end{pmatrix} = \begin{pmatrix} -5. \\ 0. \end{pmatrix}. \tag{56}$$

The external disturbance vector is as shown in (46). The component of Ax_{sp} lying in $R(B)^{\perp}$ is found to be

$$\bar{a} = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 0.2 \end{bmatrix} \begin{pmatrix} -5. \\ 0. \end{pmatrix} = \begin{pmatrix} -4. \\ 2. \end{pmatrix}, \tag{57}$$

and the component of Fw in $R(B)^{\perp}$ is as given by (49). The geometry of the disturbance vectors and the components in $R(B)^{\perp}$ for this case are shown in Figure 21.

As can be seen from Figure 21, the uncancellable components of the set-point and external disturbances act in opposite directions, unlike the situation which existed in the first example, and one would expect w to exhibit a positive utility. Upon substituting from (49) and (47) into (52), one obtains

$$\left\| \bar{a} + \bar{f} \right\|^2 - \left\| \bar{a} \right\|^2 = 5. - 20. = -15. < 0. \tag{58}$$

and it is evident from Figure 21 that $\Theta = 180°$; therefore, w does satisfy the conditions for positive utility.

The bound on the all[illegible]le magnitude of $\bar{f}$ for positive utility is

$$\left\| \bar{f} \right\| < \left\| 2\bar{a} \right\| = 8.944, \tag{59}$$

so,

$$\left\| z \right\| < 20. \tag{60}$$

For $U > 0.$, with $x_{sp} = (-5.,0.)^T$, one must have $90° < \Theta < 270°$ and $\left\| z \right\| < 20.$ Figure 22 shows the region of positive utility in the state space. Figure 23 shows the error, as a function of external disturbance, with and without disturbance minimization control. Figure 24 shows an expanded scale plot of the portion of Figure 23 between $w = 0.$ and $w = 10.$ Figure 25 shows the percent reduction in ϵ_{ss} obtained by use of the disturbance minimization controllers over the case when $u_d = 0.$, and Figure 26 is an expanded scale plot of the portion of Figure 25 between $w = 0.$ and $w = 10.$

As can be seen from Figure 24 there is again a region for w in which the controllers designed to minimize disturbance effects on $\dot{\epsilon}$ cause larger ϵ_{ss} than would $u_d \equiv 0.$ There is also a value of w, with $u_d = 0.$, which equals the performance of the controllers designed to give the minimum ϵ_{ss}.

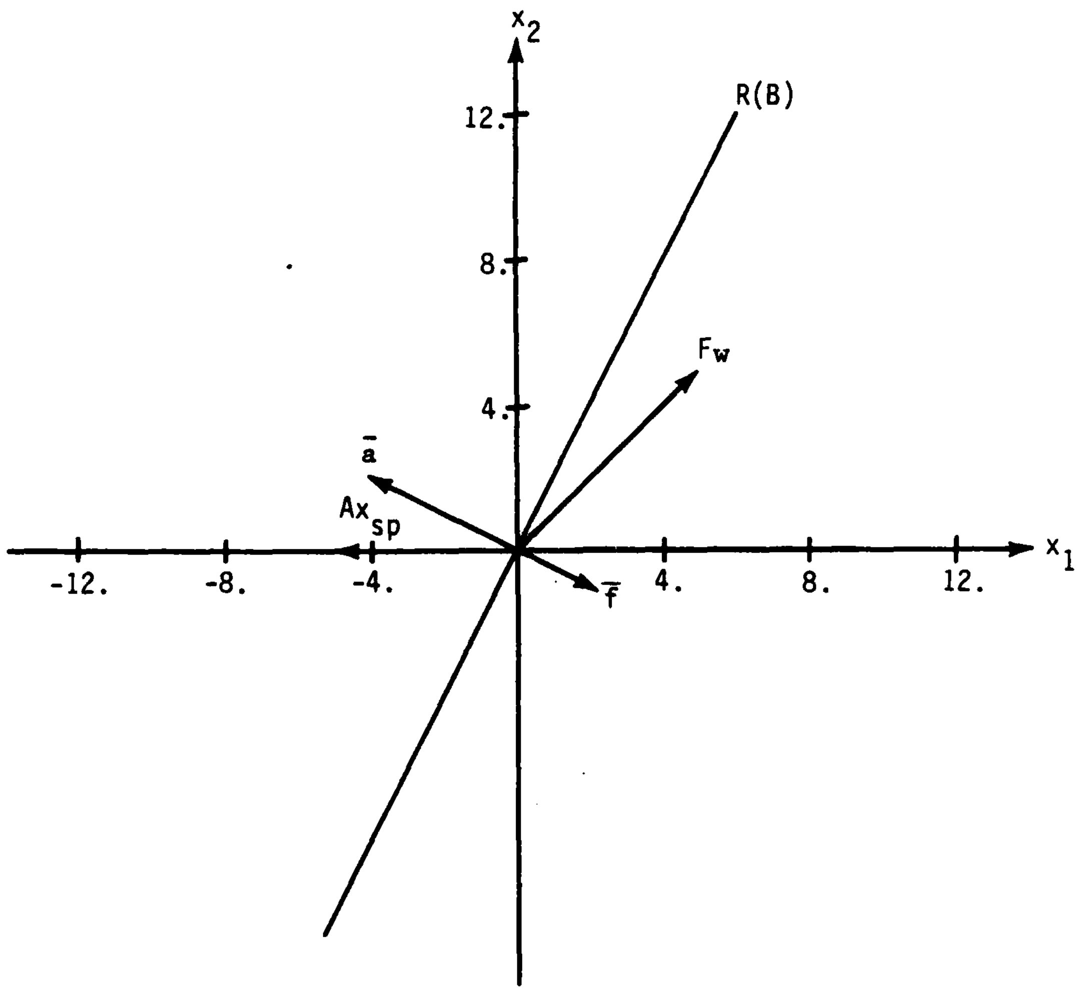

Figure 21. Geometry of disturbances relative to the line of action of the control, $x_{sp} = (-5.,0.)^T$.

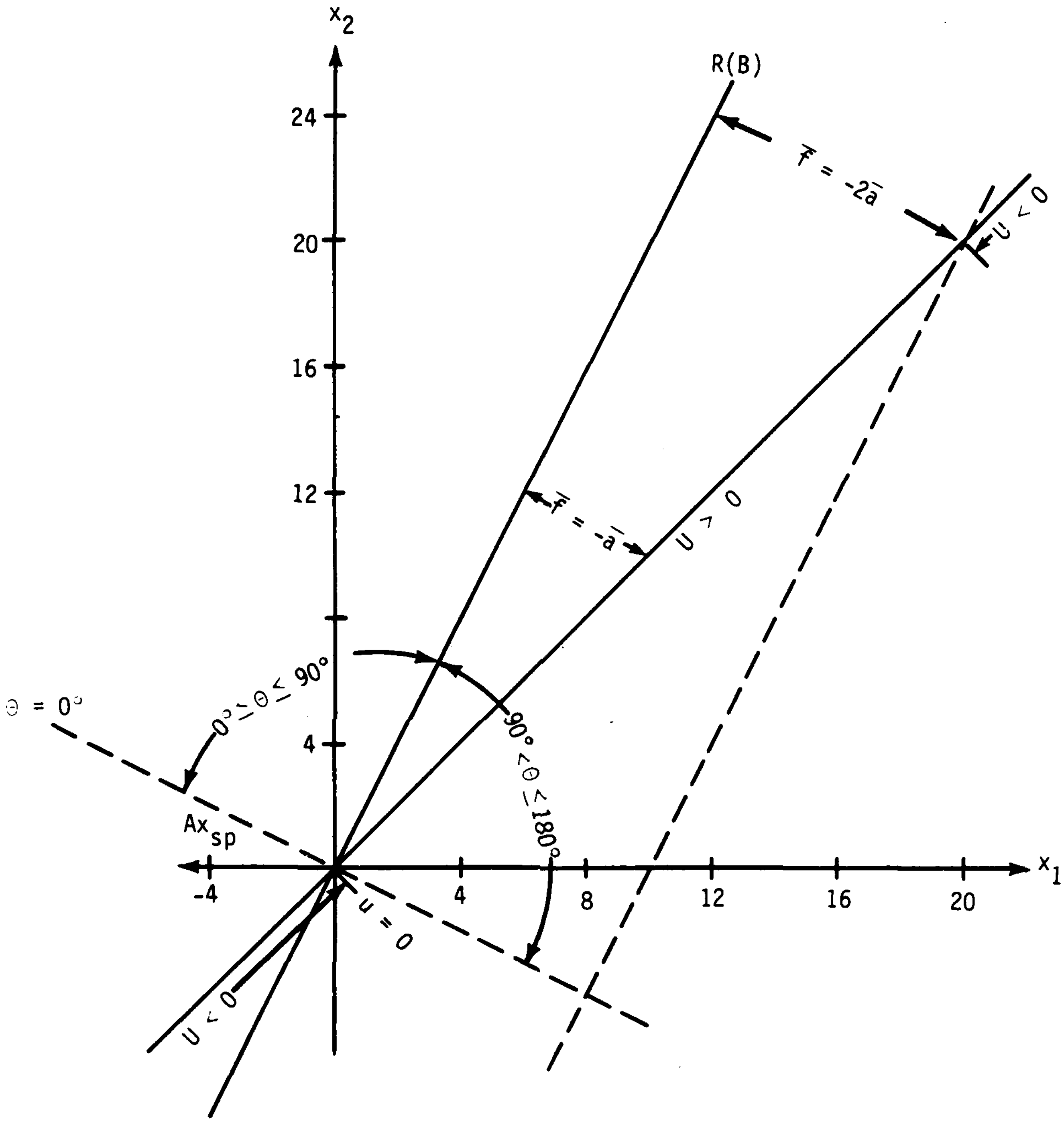

Figure 22. Region of positive utility for case with $x_{sp} = (-5.,0.)^T$.

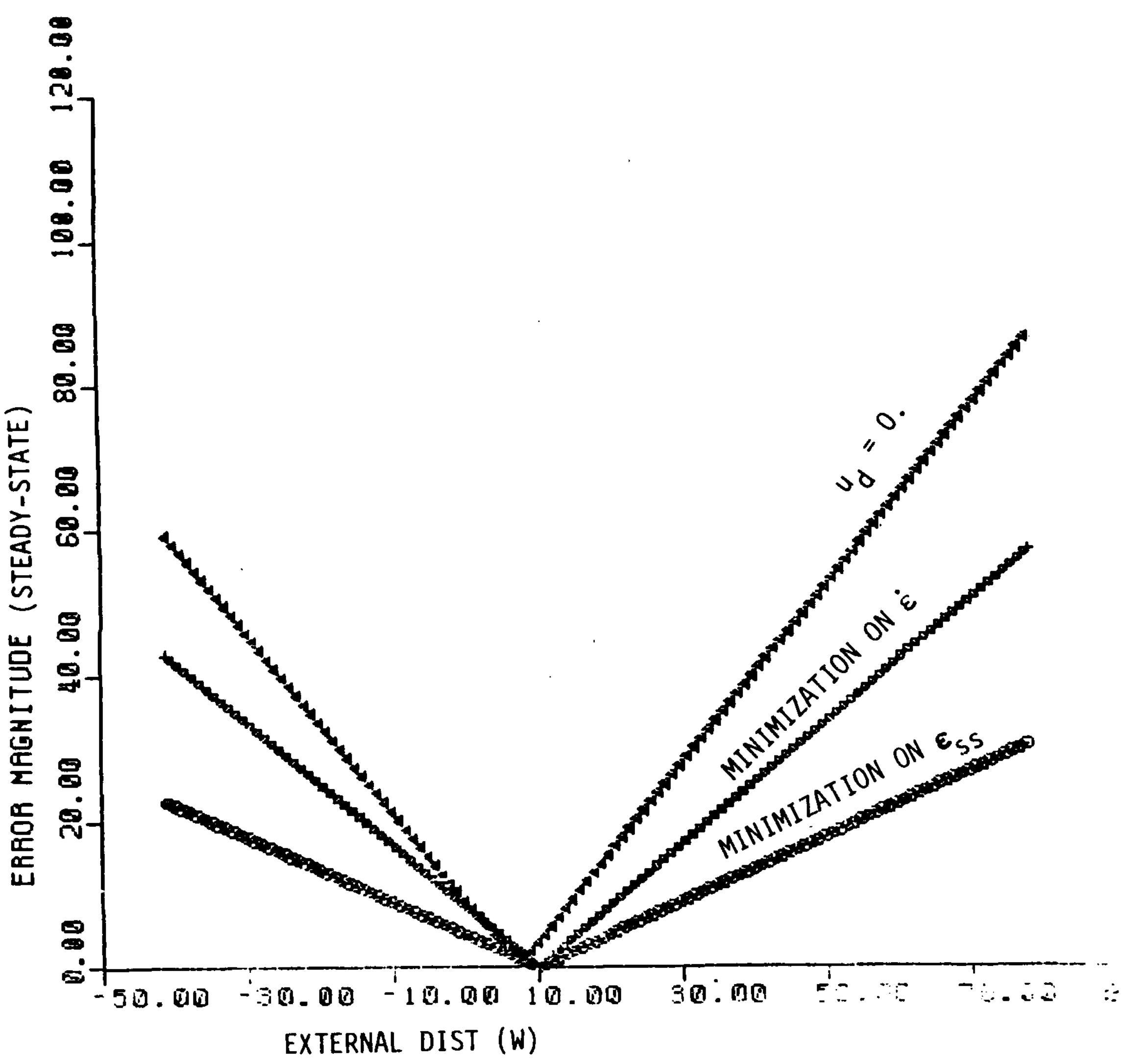

Figure 23. Steady-state error as a function of external disturbance magnitude with $x_{sp} = (-5.,0.)^T$.

```
      COMMON X(14),DX(14),KUTTA,DT,NX
      DIMENSION XDAT(50)
      NVAR=6
      WRITE(20)NVAR
      W=-42.
      DO 5050 IJ=1,120
      W=W+1.
      DO 100 I=1,14
      DX(I)=0.
      X(I)=0.
  100 CONTINUE
      TIME=0.
      XSP=-5.
      NX=16
      X1=5.
      X3=5.
C ***** INITIAL CONDITIONS ON PLANT STATES *****
      X(1)=X1
      X(3)=X1
      X(5)=X1
      X(7)=X1
      X(9)=X3
      X(11)=X3
      X(13)=X3
C ***********************************************
      DT=0.05
      UDS1S=0.
      UDW1S=0.
      UDS2S=0.
      UDW2S=0.
      UD1T=0.
      UD2T=0.
      IPRT=0
 1000 CONTINUE
      IF(TIME.GE.10.) GO TO 9999
      IPRT=IPRT+1
      DO 200 KUTTA=1,4
C ***** DIST MINIMIZING CONTROL VECTORS FOR MIN NORM OF *****
C ***** STEADY-STATE ERROR                              *****
      UDS1=-2.3447*XSP
      UDW1=-1.6723*W
C ***********************************************************
C ***** DIST MINIMIZING CONTROL VECTORS FOR MIN DISTUR- *****
C ***** BANCE CONTRIBUTION IN DIFFERENTIAL EQ FOR ERROR *****
      UDS2=-0.2*XSP
      UDW2=-0.6*W
C ***********************************************************
C ***** PLANT DIFF EQS WITH UD TO MIN NORM OF STEADY-   *****
C ***** STATE ERROR                                     *****
      DX(1)=-2.*X(1)+0.64*X(2)-XSP-UDS1-W-UDW1
      DX(2)=-6.*X(1)+0.28*X(2)-2.*UDS1-W-2.*UDW1
C ***********************************************************
C ***** PLANT DIFF EQS WITH UD=0                        *****
      DX(3)=-2.*X(3)+0.64*X(4)-XSP-W
      DX(4)=-6.*X(3)+0.28*X(4)-W
C ***********************************************************
C ***** PLANT DIFF EQS WITH UDW=0                       *****
```

APPENDIX

DIGITAL SIMULATION

REFERENCES

1. Johnson, C. D., "Accommodation of Disturbances in Optimal Control Problems," International Journal of Control, Vol. 15, No. 2, 1972, pp. 209-213 .

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4. ---"Disturbance-Accommodation Control: An Overview of the Subject," Journal of Interdisciplinary Modeling and Simulation, 3(1), 1980, pp. 1-29.

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IX. SUMMARY

It was shown in Section VI that u_d designed to minimize steady-state error does result in a smaller ε_{ss} than does u_d designed to minimize disturbance effects on $\dot{\varepsilon}$. However, neither design exhibited a consistent advantage in reducing transient excursions in the plant response.

Two examples were worked out in Section VII in order to illustrate the geometry associated with disturbance minimization for a second order state set-point regulation problem and the regions of positive and negative utility. These examples demonstrate the fact that it is possible for an uncancellable external disturbance to assist in reducing the set-point error. From Figures 17 and 23, it can be seen that this is true whether or not the disturbance minimizing control is present. The disturbance minimizing control did provide a smaller achievable steady-state error and did extend the range of external disturbances for which a positive utility could be obtained. This can be seen as illustrated in Figure 17. For the case with $u_d = 0$. the steady-state error for $-30. \leq w < 0.$ was less than or equal to the steady-state error for $w = 0.$ However, for the case where u_d was designed to minimize ε_{ss}, the range of values of w for which $U>0$ goes to -40. It is also apparent from Figures 17 and 23 that application of the disturbance minimization controller can result in zero steady-state error, for a particular value of w in each case, i.e., $w = -2X_{sp,1}$. This result can be verified from manipulation of Equation (23).

In Section VIII, it was shown that it is possible to reduce control energy expenditure in cases where w provides a steady-state error equal to that given by u_d designed to reduce ε_{ss}.

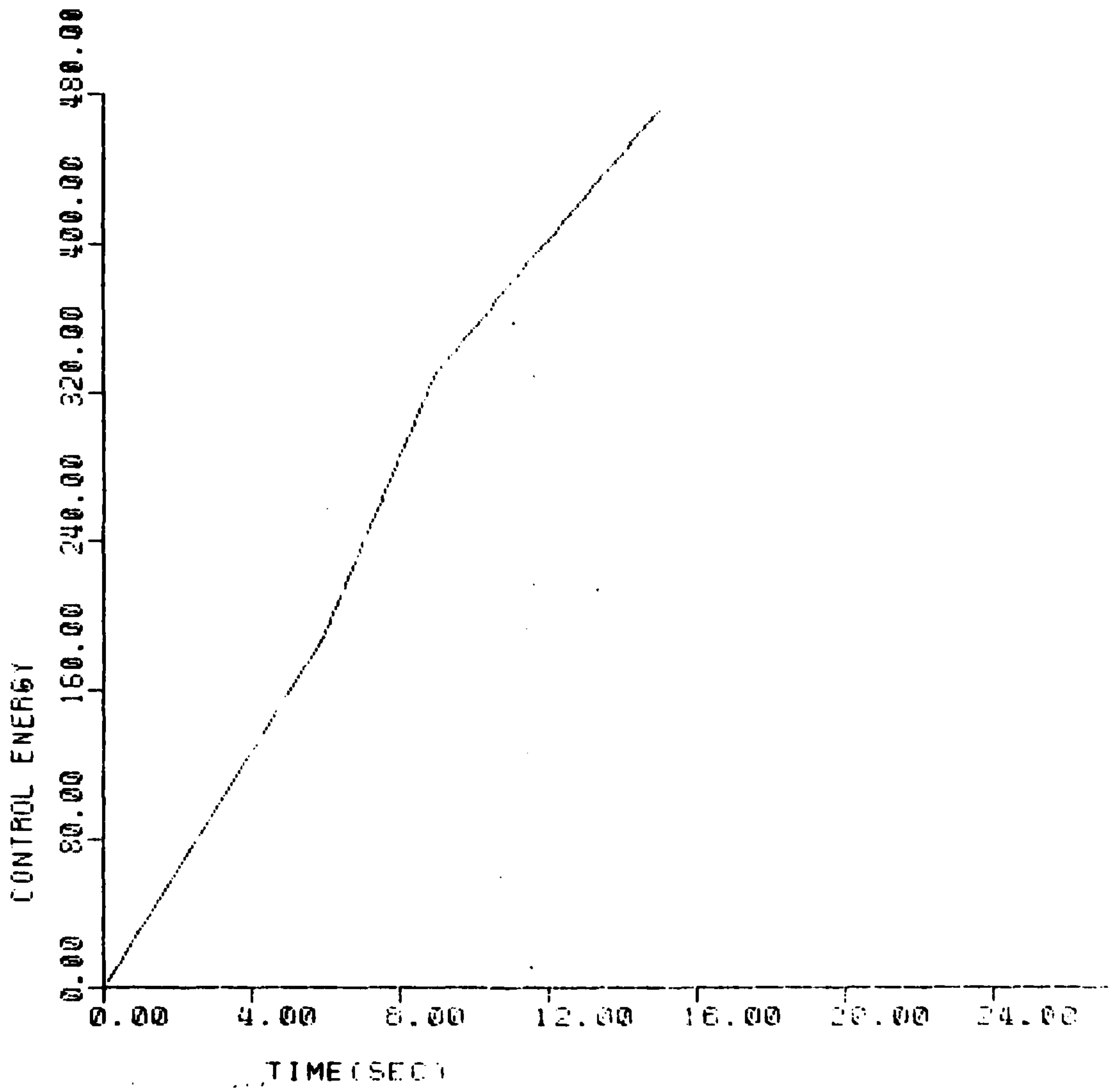
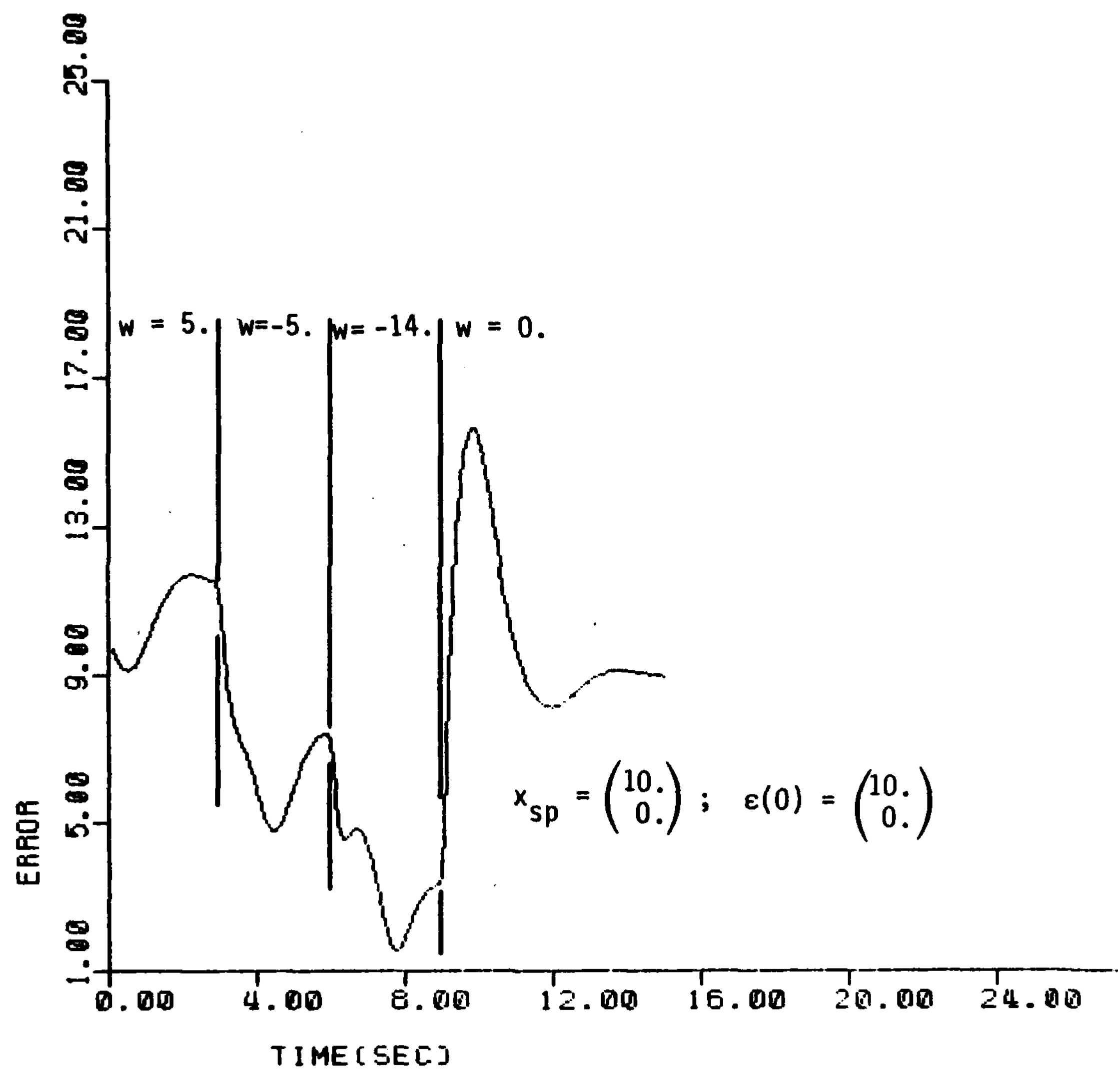


Figure 31. Control energy vs. time, corresponds to case in Figure 30.



$$x_{sp} = \begin{pmatrix} 10. \\ 0. \end{pmatrix} ; \quad \varepsilon(0) = \begin{pmatrix} 10. \\ 0. \end{pmatrix}$$

Figure 30. Set-point error for various w's, with $u_d = u_d^*$.

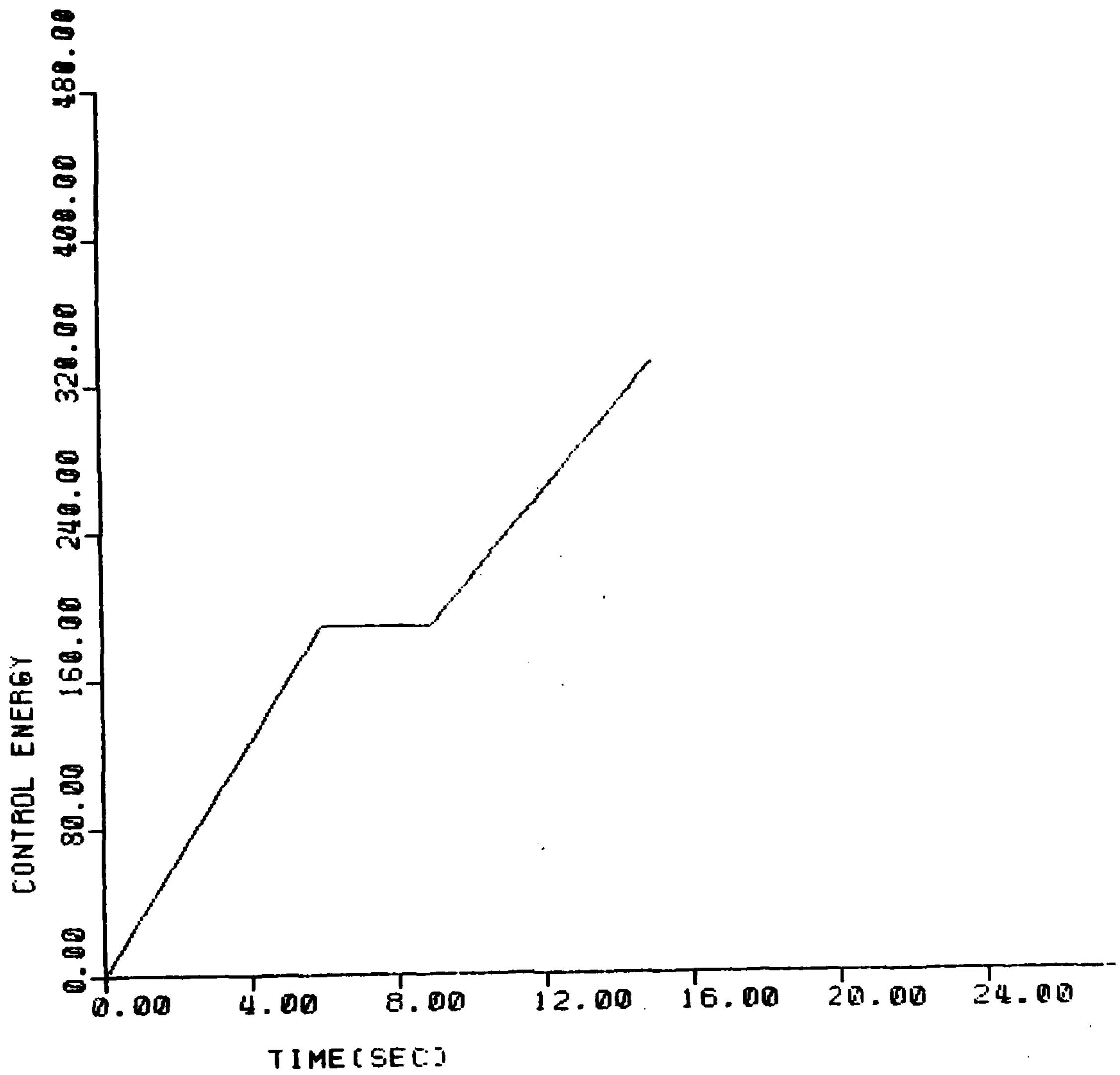


Figure 29. Control energy vs. time, corresponds to case in Figure 28.

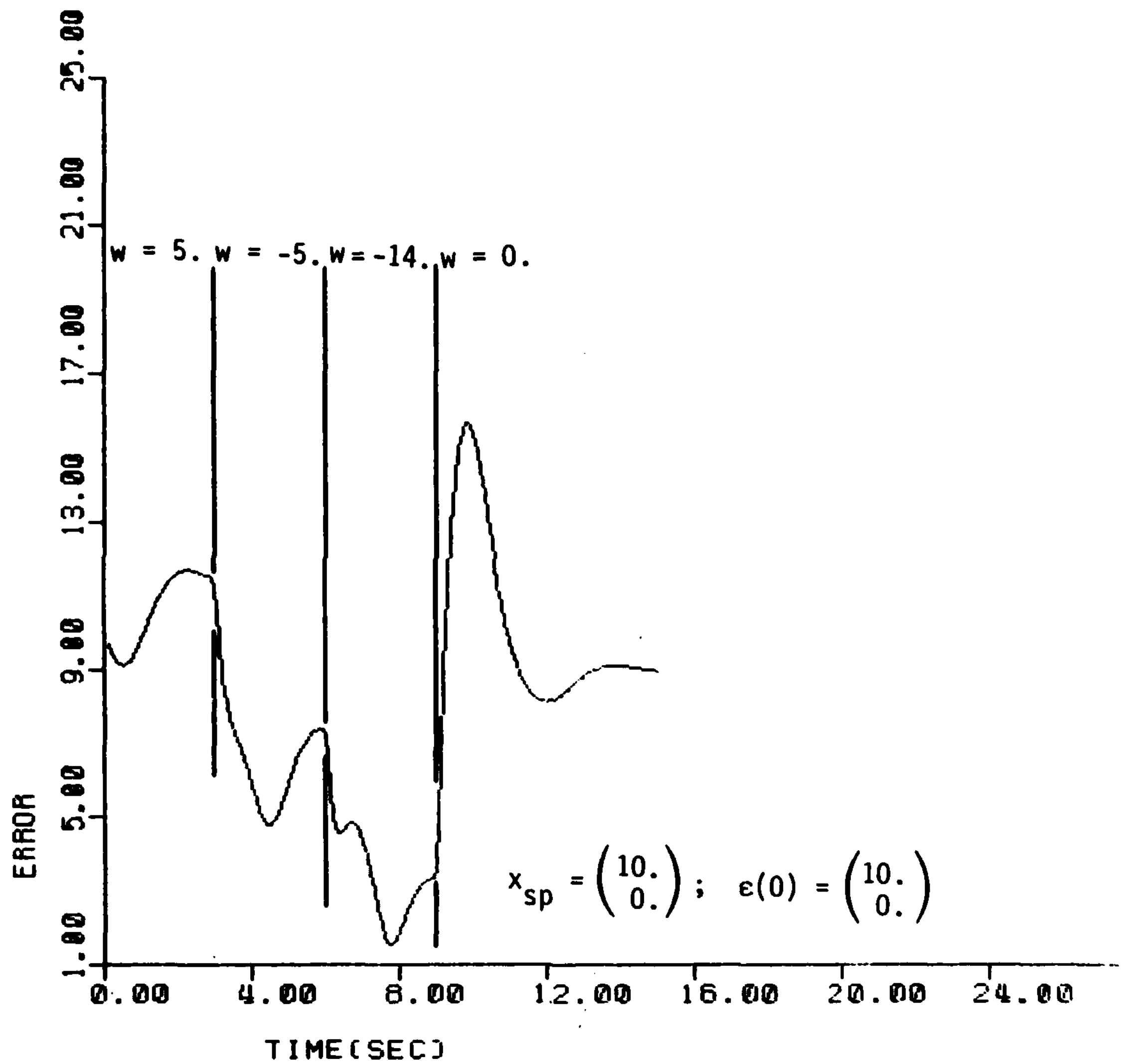


Figure 28. Set-point error for various w's, with $u_d = 0$ when $w = -14$.

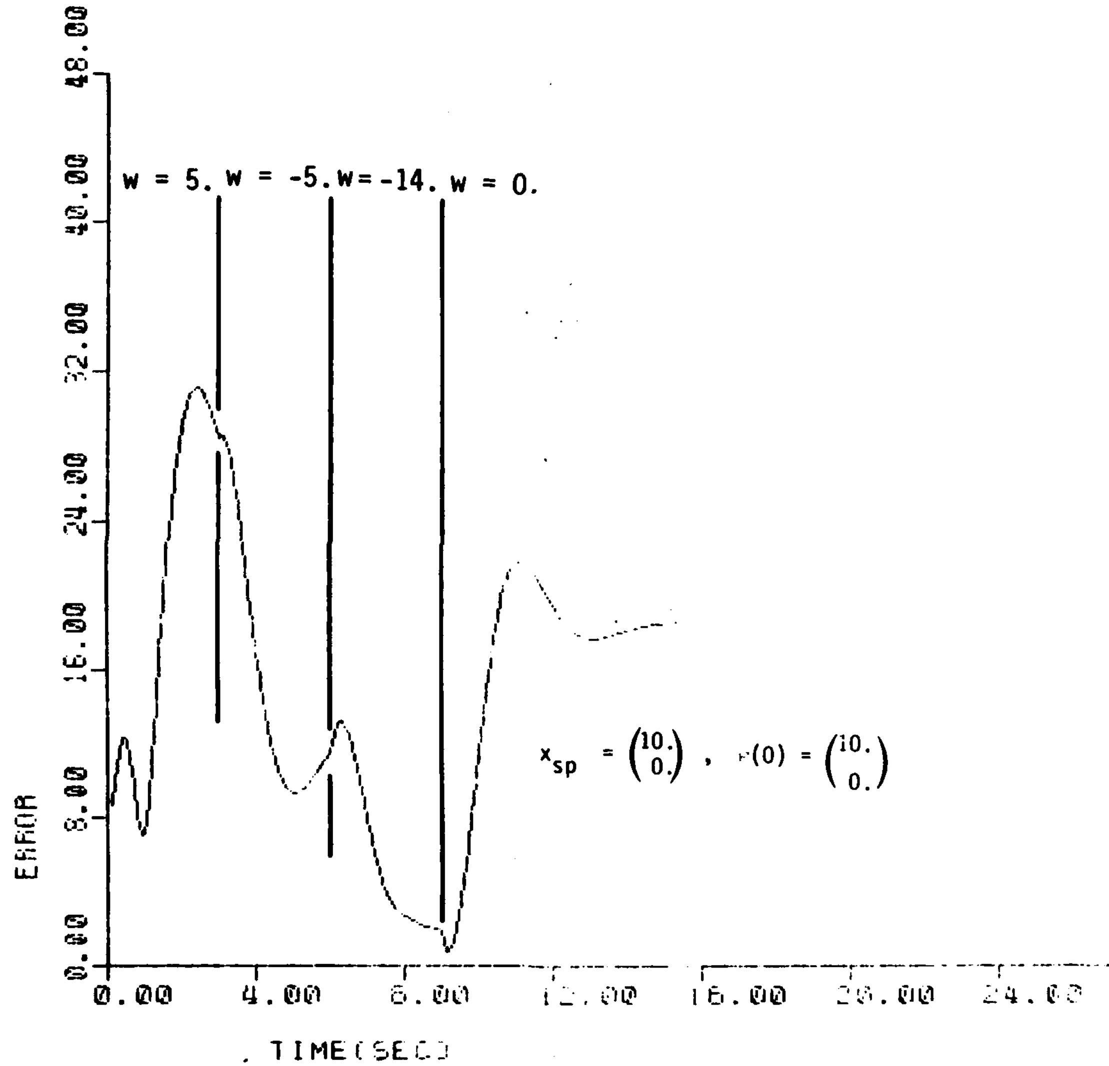


Figure 27. Set-point error for various w's, with $u_d \equiv 0$.

To illustrate use of this property, Example 1 was simulated with the magnitude of w changing at three second intervals up to a time of 9 seconds. Figure 27 shows the set-point error obtained when no disturbance minimization control is applied. Figure 28 shows the set-point error obtained when $u_d = u_d^*$ except during the interval when $w=-14.$, at which time u_d is set to zero. The control energy corresponding to this case is shown in Figure 29. Figure 30 shows the set-point error when $u_d = u_d^*$ for all values of w and Figure 31 shows the corresponding control energy. A comparison of Figures 28 and 30 shows that the two curves are identical. A comparison of Figures 29 and 31 shows that the control energy expenditure is reduced by about 140 units when u_d is set to zero for $w=-14$. Of course, these results were obtained under the assumption that w is known exactly and that the time of occurance of each change in w is known exactly, but they serve to illustrate the possibilities.

VIII. REDUCING CONTROL ENERGY EXPENDITURE

As can be seen in Figures 18 and 24, there is one point where the steady-state set-point error is the same without u_d as it is with u_d designed to minimize ε_{ss}. That is, for a given target set-point there is an external disturbance w for which the controller u_d could be set to zero without any performance degradation. There is also a small interval about this particular w within which w could be located and for which u_d would contribute little to further error reduction. This would permit u_d to be set to zero, in order to conserve control energy, without deterimental effects on performance. Note that the particular w is close to, but not coincident with, the point of minimum ε_{ss} for the u_d=0. curve in each case.

In order to determine the relationship between w and x_{sp} at these equal performance points for the plant and disturbance models of Section VII, one would equate the norms of Equation (23) and the equation resulting from (18) when $u_{ds} = u_{dw} = 0.$, i.e., the norms of

$$\varepsilon^*_{ss} = [I - (\tilde{A}^{-1}B)(\tilde{A}^{-1}B)^{\#}]\tilde{A}^{-1}(Ax_{sp} + Fw) \tag{23}$$

and

$$\varepsilon_{ss} = \tilde{A}^{-1}(Ax_{sp} + Fw). \tag{61}$$

The norm squared of (23) is given by

$$\left\|\varepsilon^*_{ss}\right\|^2 = 0.8x^2_{sp,1} + 0.8x_{sp,1}w + 0.2w^2 \tag{62}$$

and the norm squared of (61) by

$$\left\|\varepsilon_{ss}\right\|^2 = 3.352x^2_{sp,1} + 4.441x_{sp,1}w + 1.498w^2. \tag{63}$$

If one equates (62) and (63), the resulting expression is

$$x^2_{sp,1} + 1.426x_{sp,1}w + 0.508w^2 = 0 \tag{64}$$

and the solution for w is given by

$$w = -1.402x_{sp,1}. \tag{65}$$

For a given target set-point of the form $x_{sp} = (x_{sp,1}, 0.)^T$, Equation (65) will give that value for w which, if it occurs, will enable the control u_d to be set to zero without loss of performance. In Example 1, the value of w from (65) is w = -14. In Example 2, the value of w from (65) is w = -7.0. It can be seen from Figures 18 and 24 that these are the correct values.

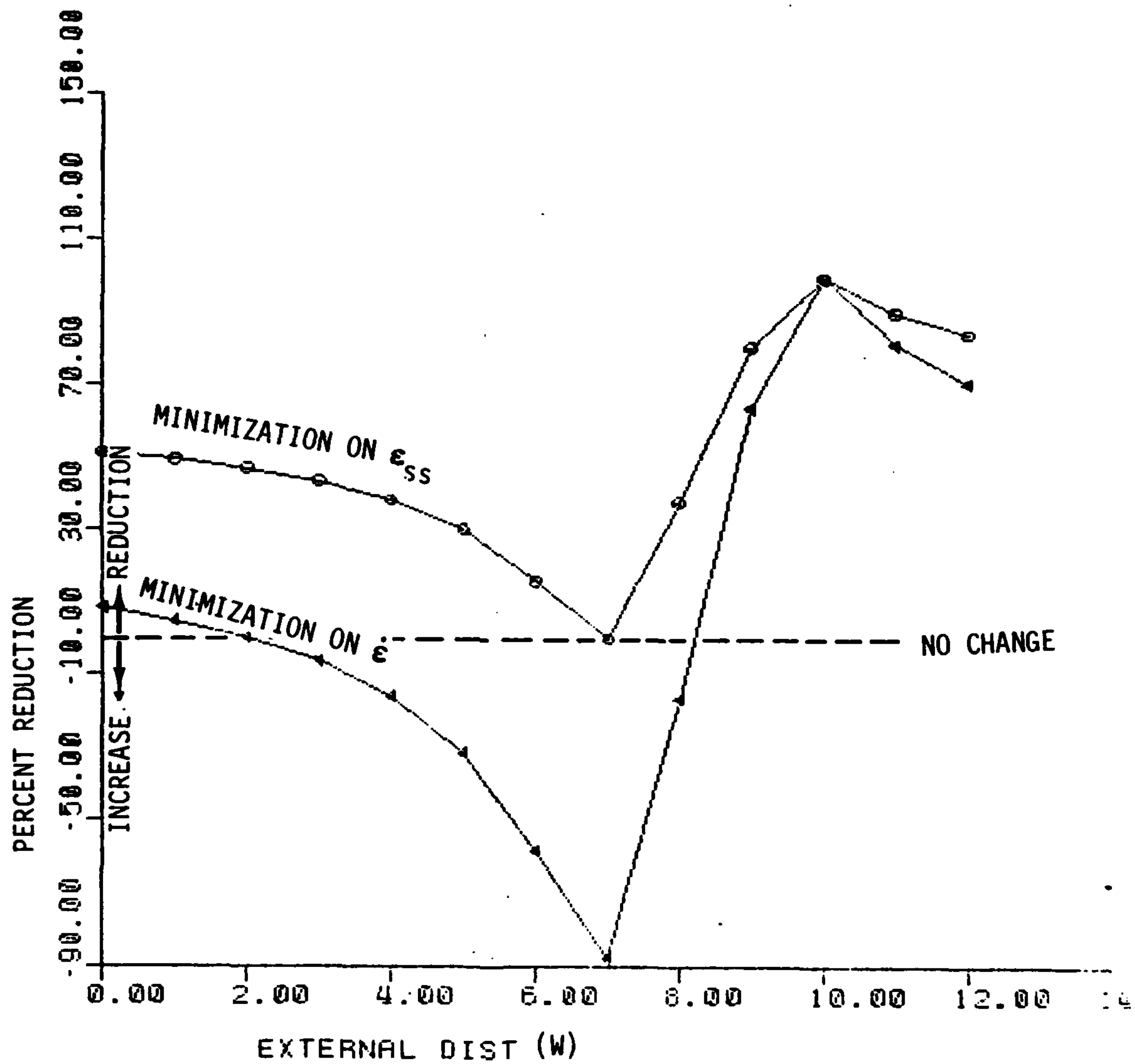


Figure 26. Expanded scale plot of a portion of Figure 25.

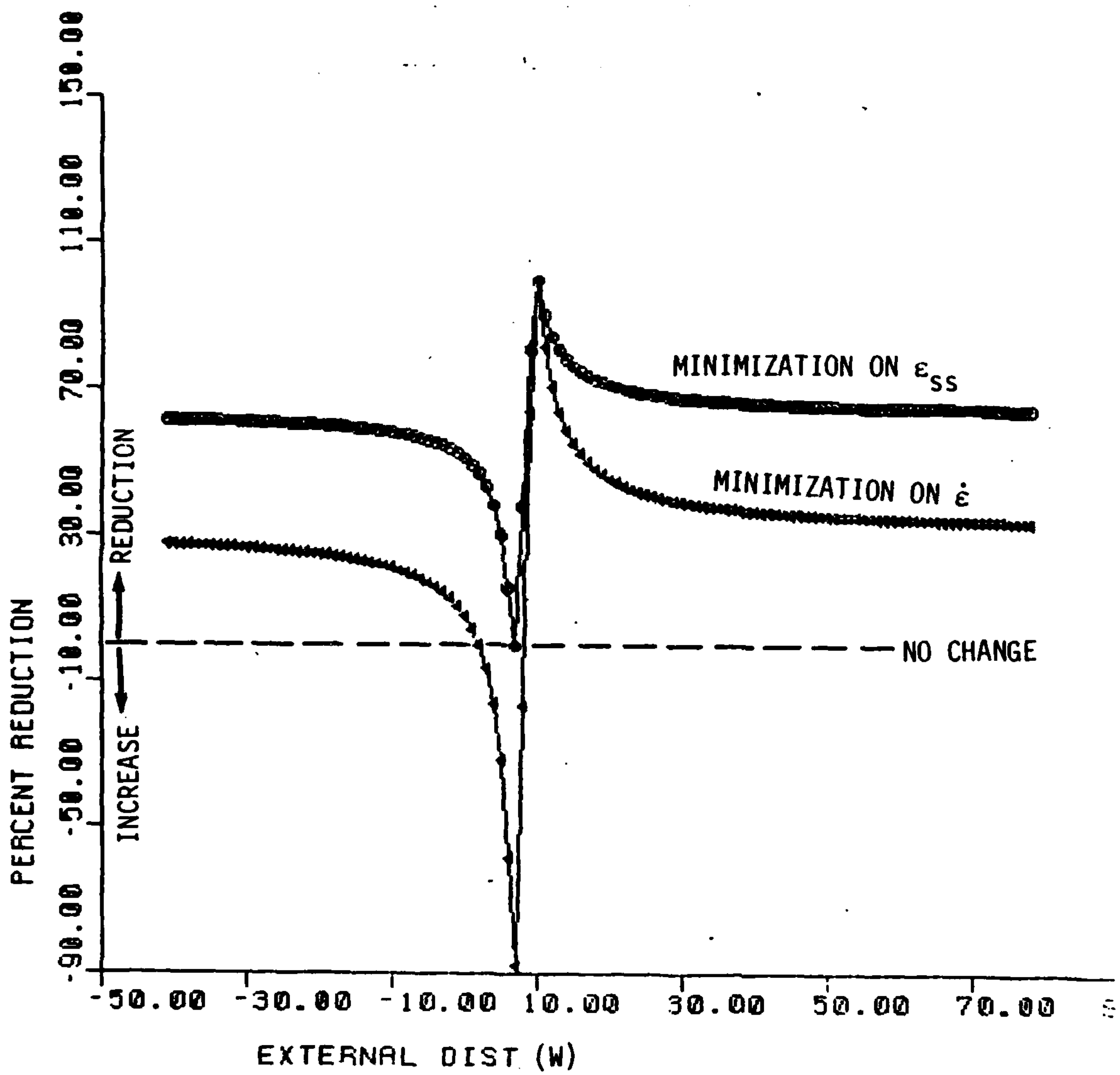


Figure 25. Percent reduction in steady-state error achieved by disturbance minimization controllers as compared to a case with $u_d = 0.$, $x_{sp} = (-5.,0.)^T$.

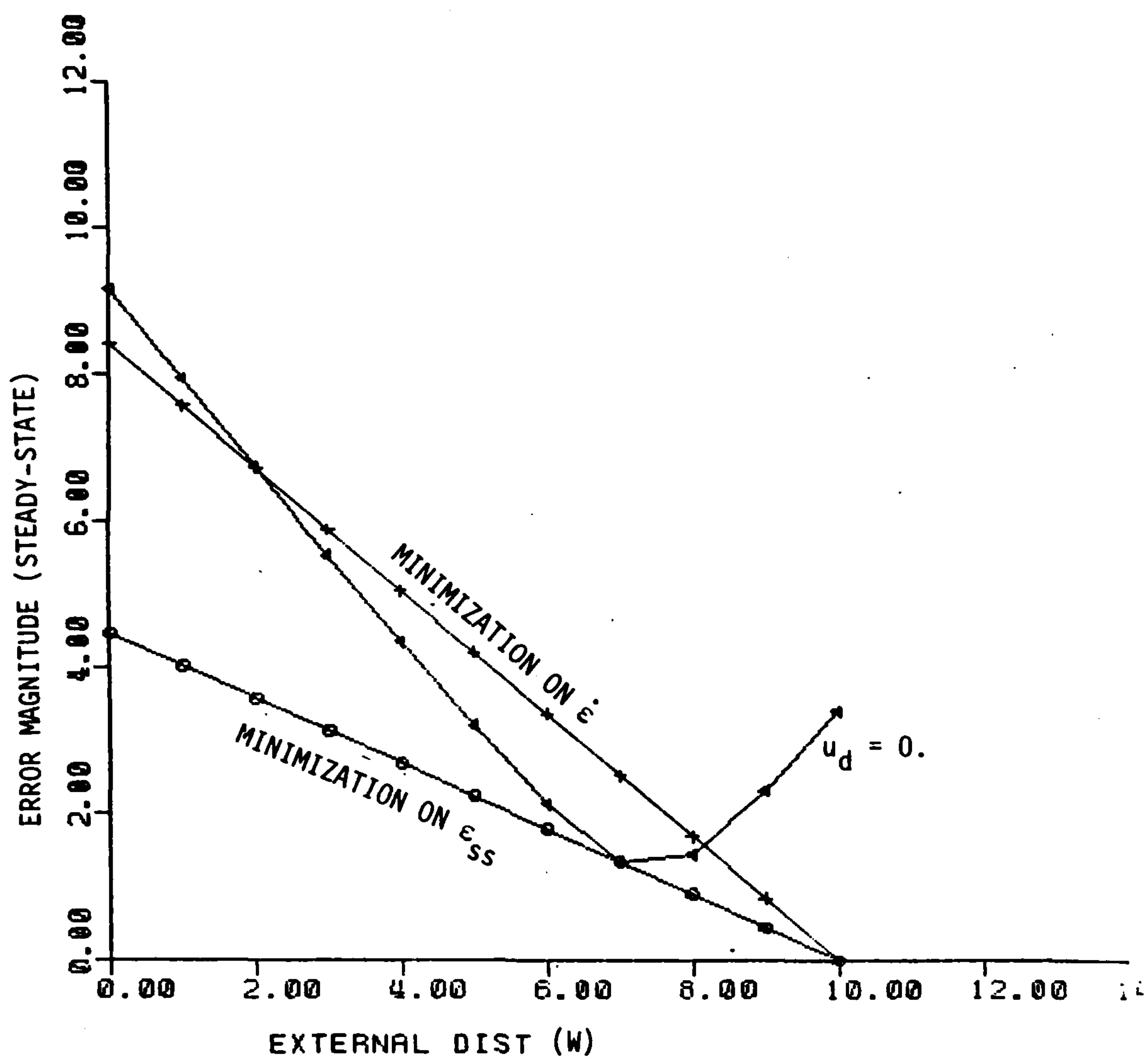


Figure 24. Expanded scale plot of a portion of Figure 23.

```
      DX(5)=-2.*X(5)+0.64*X(6)-XSP-W-UDS1
      DX(6)=-6.*X(5)+0.28*X(6)-W-2.*UDS1
C ***********************************************************
C ***** PLANT DIFF EQS WITH UDS=0.                      *****
      DX(7)=-2.*X(7)+0.64*X(8)-XSP-W-UDW1
      DX(8)=-6.*X(7)+0-0.28*X(8)-W-2.*UDW1
C ***********************************************************
C ***** PLANT DIFF EQS WITH UD TO MIN DIST CONTRIBUTION *****
C ***** IN DIFF EQS FOR ERROR                           *****
      DX(9)=-2.*X(9)+0.65*X(10)-XSP-UDS2-W-UDW2
      DX(10)=-6.*X(9)+0.28*X(10)-W-2.*UDS2-2.*UDW2
C ***********************************************************
C ***** PLANT DIFF EQS WITH UDW=0.                      *****
      DX(11)=-2.*X(13)+0.64*X(14)-XSP-W-UDS2
      DX(12)=-6.*X(13)+0.28*X(14)-W-2.*UDS2
C ***********************************************************
C ***** PLANT DIFF EQS WITH UDS=0                       *****
      DX(13)=-2.*X(15)+0.64*X(16)-XSP-W-UDW2
      DX(14)=-6.*X(15)+0.28*X(16)-W-2.*UDW2
C ***********************************************************
      GO TO (30,60,30,40),KUTTA
   30 CONTINUE
      TIME=TIME+.5*DT
   40 CONTINUE
   60 CALL RUNK
  200 CONTINUE
C ***** MEASURE OF CONTROL ENERGY                       *****
      UDS1S=UDS1S+ABS(UDS1*DT)
      UDW1S=UDW1S+ABS(UDW1*DT)
      UDS2S=UDS2S+ABS(UDS2*DT)
      UDW2S=UDW2S+ABS(UDW2*DT)
      UD1T=UDS1S+UDW1S
      UD2T=UDS2S+UDW2S
C ***********************************************************
C ***** ERROR MAGNITUDE IN EACH CASE                    *****
      ERR1=SQRT(X(1)**2+X(2)**2)
      ERR2=SQRT(X(3)**2+X(4)**2)
      ERR3=SQRT(X(5)**2+X(6)**2)
      ERR4=SQRT(X(7)**2+X(8)**2)
      ERR5=SQRT(X(9)**2+X(10)**2)
      ERR6=SQRT(X(11)**2+X(12)**2)
      ERR7=SQRT(X(13)**2+X(14)**2)
C ***********************************************************
      IF(IPRT.NE.2) GO TO 500
      IPRT=0
  500 GO TO 1000
   50 FORMAT ('  TIME  ',F8.4,'  ERR1  ',F8.4,'  ERR2  ',F8.4,
     .'  ERR3  ',F8.4,'  ERR4  ',F8.4,'  ERR5  ',F8.4)
 9999 CONTINUE
C ***** PERCENT REDUCTION FROM CASE WITH UD=0           *****
      XPCES=((ERR2-ERR1)/ERR2)*100.
      XPCED=((ERR2-ERR5)/ERR2)*100.
C ***********************************************************
C ***** PLOT VARIABLES                                  *****
      XDAT(1)=W
      XDAT(2)=XPCES
      XDAT(3)=XPCED
```

```
            XDAT(4)=ERR1
            XDAT(5)=ERR2
            XDAT(6)=ERR5
      C ****************************************************************
            WRITE(20) (XDAT(I),I=1,NVAR)
            PRINT *, IJ,W
 5050       CONTINUE
            STOP
            END
```

PROGRAM SECTIONS

	Name	Bytes	Attributes						
0	$CODE	1118	PIC	CON	REL	LCL	SHR	EXE	RD
2	$LOCAL	324	PIC	CON	REL	LCL	NOSHR	NOEXE	RD
3	$BLANK	124	PIC	OVR	REL	GBL	SHR	NOEXE	RD
	Total Space Allocated	1566							

ENTRY POINTS

Address	Type	Name
0-00000000		DMPLIS$MAIN

VARIABLES

Address	Type	Name	Address	Type	Name	Address	Ty
3-00000074	R*4	DT	2-00000114	R*4	ERR1	2-00000118	R
2-00000120	R*4	ERR4	2-00000124	R*4	ERR5	2-00000128	R
2-000000D4	I*4	I	2-000000D0	I*4	IJ	2-00000100	I
2-000000C8	I*4	NVAR	3-00000078	I*4	NX	2-000000D8	R
2-000000FC	R*4	UD2T	2-00000104	R*4	UDS1	2-000000E8	R
2-000000F0	R*4	UDS2S	2-00000108	R*4	UDW1	2-000000EC	R
2-000000F4	R*4	UDW2S	2-000000CC	R*4	W	2-000000E0	R
2-00000134	R*4	XPCED	2-00000130	R*4	XPCES	2-000000DC	R

ARRAYS

Address	Type	Name	Bytes	Dimensions
3-00000038	R*4	DX	56	(14)
3-00000000	R*4	X	56	(14)
2-00000000	R*4	XDAT	200	(50)

```
      SUBROUTINE RUNK
      COMMON X(14),DX(14),KUTTA,DT,NX
      DIMENSION XA(14),DXA(14)
      GO TO (10,30,50,70),KUTTA
10    DO 20 I=1,NX
      XA(I)=X(I)
      DXA(I)=DT*DX(I)
20    X(I)=X(I)+.5*DXA(I)
      RETURN
30    TDT=2.*DT
      HDT=.5*DT
      DO 40 I=1,NX
      DXA(I)=DXA(I)+TDT*DX(I)
40    X(I)=XA(I)+HDT*DX(I)
      RETURN
50    DO 60 I=1,NX
      VDT=DT*DX(I)
      DXA(I)=DXA(I)+2.*VDT
60    X(I)=XA(I)+VDT
      RETURN
70    DO 80 I=1,NX
80    X(I)=XA(I)+(DXA(I)+DT*DX(I))/6.
      RETURN
      END
```

PROGRAM SECTIONS

	Name	Bytes	Attributes						
0	$CODE	242	PIC	CON	REL	LCL	SHR	EXE	RD
2	$LOCAL	128	PIC	CON	REL	LCL	NOSHR	NOEXE	RD
3	$BLANK	124	PIC	OVR	REL	GBL	SHR	NOEXE	RD
	Total Space Allocated	494							

ENTRY POINTS

Address	Type	Name
0-00000000		RUNK

VARIABLES

Address	Type	Name	Address	Type	Name	Address	Ty
3-00000074	R*4	DT	2-00000078	R*4	HDT	2-00000070	I
3-00000078	I*4	NX	2-00000074	R*4	TDT	2-0000007C	F

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END

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